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PREFACE

The workshop **" What comes beyond the standard model? "** was meant as a real workshop in which participants would spend most of the time in discussions, confronting different aproaches and ideas. The nice town of Bled by the lake of the same name, surrounded by beautiful mountains and offering pleasant walks, was chosen to stimulate the discussions.

We believe that this really happened. We spent ten fruitful days in the house of Josip Plemelj, which belongs to the Physical Society and Mathematical Society of Slovenia, discussing the open problems in high energy physics.

We tried to answer to some of the open questions which the electroweak Standard Model leaves unanswered. We started the first day of the workshop with the following list of open questions:

- Are spins and charges unified ? How can one otherwise understand the connection between spin and weak charge, built into the Standard Model by the following requirement: there exist only left handed weak charge doublets and right handed weak charge singlets, which assures parity violation ?
- Why is parity not broken in strong and electromagnetic interactions?
- How is parity conserved in broken $SO(10)$?
- A Majorana particle, if it exists at all, is a rather unusual particle, since it is a particle and its own antiparticle at the same time, like a photon, but it is a fermion. Can one formulate Majorana quantum field theory so that one sees the Dirac sea being filled by Majorana particles? Can we make a Majorana field theory starting from the zero mass Weyl theory, then adding a mass term as an interaction in a similar way as we can do in the case of a Dirac particle? What is the Majorana propagator?
- How one can understand the hierarchy problem and the scale problem?
- Where do the generations of quarks and leptons come from? Do we have more than three generations? Is there any approach which would suggest more than three generations and would not contradict the experimental data?
- Where do the Yukawa couplings come from ?
- Can we exclude the existence of constituents of quarks and leptons? Shall we not have problems with confining chiral particles into chiral clusters of particles?
- Is $S(U(2)\times U(3))$ a subgroup of (i) $SO(1,13)$, (ii)$SU(4)_C\times SU(2)_L\times SU(2)_R$, (iii) other unified gauge groups?
- Can spin and charges be unified within $SO(1,d-1)$?
- Can one connect the Joos representation of Dirac spinors and spinor representations in Grassmann space?

We have tried for 10 days to answer these questions. The result of this effort is collected in the Proceedings. We, of course, only succeeded to partly answer some of these questions. Everybody tried to answer the questions connected with her/his own work. Because of that the Proceedings contains a review of previous work, connected with the above listed questions.
Next year we shall meet again at Bled.

The organizers are grateful to all the participants for the real discussing and working atmosphere.

The organizers would like to thank Anamarija Borštnik, Jiannis Pachos, and Andreja Šarlah for the efficient help in the organization of the workshop and in the preparation of the proceedings.

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Invited Talks

The Problem of the Quark-Lepton Mass Spectrum

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1 Introduction

The charged fermion masses and mixing angles arise from the Yukawa couplings, which are arbitrary parameters in the Standard Model (SM). The masses range over five orders of magnitude, from 1/2 MeV for the electron to 175 GeV for the top quark. Also the elements of the quark weak coupling matrix, V_{CKM}, range from $V_{ub} \simeq 0.003$ to $V_{tb} \simeq 1$. This constitutes the charged fermion mass and mixing problem. It is only the top quark which has a mass of the order of the electroweak scale $< \phi_{WS} >= 174$ GeV and a Yukawa coupling constant of order unity $y_t \simeq 1$. It therefore seems likely that the top quark mass will be understood dynamically before those of the other fermions. All of the other Yukawa couplings are suppressed, suggesting the existence of physics beyond the SM. Furthermore the accumulating evidence for neutrino oscillations provides direct evidence for physics beyond the SM, in the form of non-zero neutrino masses.

A fermion mass term essentially represents a transition amplitude between a left-handed Weyl field ψ_L and a right-handed Weyl field ψ_R. If ψ_L and ψ_R have different quantum numbers, i.e. belong to inequivalent irreducible representations of a symmetry group G (G is then called a chiral symmetry), the mass term is forbidden in the limit of exact G symmetry and they represent two massless Weyl particles. G thus "protects" the fermion from getting a mass. For example the $SU(2)_L \times U(1)$ gauge quantum numbers of the left and right-handed top quark fields are different and the electroweak gauge symmetry protects the top quark from having a mass, i.e. the mass term $\overline{t}_L t_R$ is not gauge invariant. It is only after the $SU(2)_L \times U(1)$ gauge symmetry is spontaneously broken that the top quark gains a mass $m_t = y_t < \phi_{WS} >$, which is consequently suppressed relative to the presumed fundamental (GUT, Planck...) mass scale M by the symmetry breaking parameter $\epsilon =< \phi_{WS} > /M$. The other quarks and leptons have masses suppressed relative to $< \phi_{WS} >$ and it is natural to assume that they are protected by further approximately conserved chiral flavour charges [1], as we discuss further in section 5.

We first consider dynamical calculations of the top quark and the Higgs particle masses, using Infra-Red Quasi-Fixed Points in section 2 and the so-called Multiple Point Principle in section 3. Mass matrix ansätze with texture zeros are considered in section 4. Finally, the neutrino mass problem is briefly discussed in section 6.

2 Top and Higgs Masses from Infra-red Fixed Point

The idea that some of the SM mass parameters might be determined as infra-red fixed point values of renormalisation group equations (RGEs) was first considered [1] some time ago. It was pointed out that the three generation fermion mass hierarchy does not naturally arise out of the general structure of the RGEs, although it does seem possible in special circumstances [2]. However it was soon [3] realised that the top quark mass might correspond to a fixed point value or more likely a quasi-fixed point [4] at the scale $\mu = m_t$.

The SM quasi-fixed point prediction of the top quark mass is based on two assumptions: (a) the perturbative SM is valid up to some high (e. g. GUT or Planck) energy scale $M \simeq 10^{15} - 10^{19}$ GeV, and (b) the top Yukawa coupling constant is large at the high scale $g_t(M) \gtrsim 1$. Neglecting the lighter quark masses and mixings, which is a good approxmation, the SM one loop RGE for the top quark running Yukawa coupling $g_t(\mu)$ is:

$$16\pi^2 \frac{dg_t}{d\ln\mu} = g_t \left(\frac{9}{2} g_t^2 - 8g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{12} g_1^2 \right) \tag{1.1}$$

Here the $g_i(\mu)$ are the three SM running gauge coupling constants. The nonlinearity of the RGEs then strongly focuses $g_t(\mu)$ at the electroweak scale to its quasi-fixed point value. The RGE for the Higgs self-coupling $\lambda(\mu)$

$$16\pi^2 \frac{d\lambda}{d\ln\mu} = 12\lambda^2 + 3\left(4g_t^2 - 3g_2^2 - g_1^2\right)\lambda + \frac{9}{4}g_2^4 + \frac{3}{2}g_2^2 g_1^2 + \frac{3}{4}g_1^4 - 12g_t^4 \tag{1.2}$$

similarly focuses $\lambda(\mu)$ towards a quasi-fixed point value, leading to the SM fixed point predictions [4] for the running top quark and Higgs masses:

$$m_t \simeq 225 \text{ GeV} \quad m_H \simeq 250 \text{ GeV} \tag{1.3}$$

Unfortunately these predictions are inconsistent with the experimental running top mass $m_t \simeq 165 \pm 6$ GeV.

The corresponding Minimal Supersymmetric Standard Model (MSSM) quasi-fixed point prediction for the running top quark mass is [5]:

$$m_t(m_t) \simeq (190 \text{ GeV}) \sin\beta \tag{1.4}$$

which is remarkably close to the experimental value for $\tan\beta = 2 \pm 0.5$. Some of the soft SUSY breaking parameters are also attracted to quasi-fixed point values [6]. For example the trilinear stop coupling $A_t(m_t) \to -0.59 m_{gluino}$. For this low $\tan\beta$ fixed point, there is an upper limit on the lightest Higgs boson mass: $m_{h_0} \lesssim 100$ GeV. There is also a high $\tan\beta = 60 \pm 5$ fixed point solution [7], corresponding to large Yukawa coupling constants for the b quark and τ lepton as well as for the t quark, sometimes referred to as the Yukawa Unification scenario. In this case the lightest Higgs boson mass is $m_{h_0} \simeq 120$ GeV. The origin of the large value of $\tan\beta$ is of course a puzzle and also SUSY radiative corrections to m_b are then generically large.

3 Top and Higgs Masses from Multiple Point Principle

According to the Multiple Point Principle (MPP) [8], Nature chooses coupling constant values such that a number of vacuum states have the same energy density. This principle was first used in the Anti-Grand Unification Model (AGUT) [9, 10], as a way of calculating the values of the SM gauge coupling constants. In the Euclidean (imaginary time) formulation, the theory has a phase transition with the phases corresponding to the degenerate vacua. The coupling constants then become dynamical, in much the same way as in baby-universe theory, and take on fine-tuned values determined by the multiple point. This fine-tuning of the coupling constants is similar to that of temperature in a microcanonical ensemble, such as a mixture of ice and water in a thermally isolated container.

Here we apply the MPP to the pure SM, which we assume valid up to the Planck scale. This implies [11] that the effective SM Higgs potential $V_{eff}(|\phi|)$ should have a second minimum degenerate with the well-known first minimum at the electroweak scale $\langle|\phi_{vac\ 1}|\rangle = 174$ GeV. Thus we predict that our vacuum is barely stable and we just lie on the vacuum stability curve in the top quark, Higgs particle mass (M_t, M_H) plane. Furthermore we expect the second minimum to be within an order of magnitude or so of the fundamental scale, i.e. $\langle|\phi_{vac\ 2}|\rangle \simeq M_{Planck}$. In this way, we essentially select a particular point on the SM vacuum stability curve and hence the MPP condition predicts precise values for the pole masses [11]:

$$M_t = 173 \pm 5 \text{ GeV} \quad M_H = 135 \pm 9 \text{ GeV} \tag{1.5}$$

4 Mass Matrix Texture and Ansätze

By imposing symmetries and texture zeros on the fermion mass matrices, it is possible to obtain testable relations between the masses and mixing angles. The best known ansatz for the quark mass matrices is due to Fritzsch [12]:

$$M_U = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix} \qquad M_D = \begin{pmatrix} 0 & C' & 0 \\ C' & 0 & B' \\ 0 & B' & A' \end{pmatrix} \tag{1.6}$$

It contains 6 complex parameters A, B, C, A', B' and C', where it is necessary to *assume*:

$$|A| \gg |B| \gg |C|, \qquad |A'| \gg |B'| \gg |C'| \tag{1.7}$$

in order to obtain a good fermion mass hierarchy. Four of the phases can be rotated away by redefining the phases of the quark fields, leaving just 8 real parameters (the magnitudes of A, B, C, A', B' and C' and two phases ϕ_1 and ϕ_2) to reproduce 6 quark masses and 4 angles parameterising V_{CKM}. There are thus two relationships predicted by the Fritzsch ansatz:

$$|V_{us}| \simeq \left|\sqrt{\frac{m_d}{m_s}} - e^{-i\phi_1}\sqrt{\frac{m_u}{m_c}}\right|, \qquad |V_{cb}| \simeq \left|\sqrt{\frac{m_s}{m_b}} - e^{-i\phi_2}\sqrt{\frac{m_c}{m_t}}\right| \tag{1.8}$$

The first prediction is a generalised version of the relation $\theta_c \simeq \sqrt{\frac{m_d}{m_s}}$ for the Cabibbo angle, which originally motivated the two generation Fritzsch ansatz and is well satisfied experimentally. However the second relationship cannot be satisfied with a heavy top quark mass $m_t > 100$ GeV and the original three generation Fritzsch ansatz is excluded by the data. Consistency with experiment can, for example, be restored by introducing a non-zero 22 mass matrix element. A systematic analysis of symmetric quark mass matrices with 5 or 6 texture zeros at the the SUSY-GUT scale M_X yields five solutions [13]. An example, in which the non-zero elements are expressed in terms of a small parameter $\epsilon = \sqrt{\frac{m_c}{m_t}} = 0.058$, is described in the Stech's talk [14].

The minimal SU(5) SUSY-GUT relation (using a Higgs field in the **5** representation) for the third generation, $m_b(M_X) = m_\tau(M_X)$, is successful. However it cannot be extended to the first two generations as it predicts $m_d/m_s = m_e/m_\mu$, which fails phenomenologically by an order of magnitude. This led Georgi and Jarlskog [15] to introduce an ad-hoc coupling of the second generation to a Higgs field in the $\overline{\mathbf{45}}$ representation, giving mass matrices with the following texture:

$$M_U = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix} \quad M_D = \begin{pmatrix} 0 & F & 0 \\ F' & E & 0 \\ 0 & 0 & D \end{pmatrix} \quad M_E = \begin{pmatrix} 0 & F & 0 \\ F' & -3E & 0 \\ 0 & 0 & D \end{pmatrix} \tag{1.9}$$

and the successful mass relation $m_d/m_s = 9m_e/m_\mu$. This ansatz has been developed further in the context of an SO(10) SUSY-GUT effective operator analysis [16] to give a good fit to all the masses and mixing angles.

5 Mass Hierarchy from Chiral Flavour Charges

As we pointed out in section 1, a natural resolution to the charged fermion mass problem is to postulate the existence of some approximately conserved chiral charges beyond the SM. These charges, which we assume to be the gauge quantum numbers in the fundamental theory beyond the SM, provide selection rules forbidding the transitions between the various left-handed and right-handed quark-lepton states, except for the top quark. In order to generate mass terms for the other fermion states, we have to introduce new Higgs fields, which break the fundamental gauge symmetry group G down to the SM group. We also need suitable intermediate fermion states to mediate the forbidden transitions, which we take to be vector-like Dirac fermions with a mass of order the fundamental scale M_F of the theory. In this way effective SM Yukawa coupling constants are generated, which are suppressed by the appropriate product of Higgs field vacuum expectation values measured in units of M_F.

Consider, for example, an $SMG \times U(1)_f$ model, obtained by extending the SM gauge group $SMG = S(U(3) \times U(2)) \simeq SU(3) \times SU(2) \times U(1)$ with a gauged abelian flavour group $U(1)_f$. $SMG \times U(1)_f$ is broken to SMG by the VEV of a scalar field ϕ_S where $\langle\phi_S\rangle < M_F$ and ϕ_S carries $U(1)_f$ charge $Q_f(\phi_S) = 1$. Suppose further that $Q_f(\phi_{WS}) = 0$, $Q_f(b_L) = 0$ and $Q_f(b_R) = 2$. Then it is natural to expect the generation of a b mass of order:

$$\left(\frac{\langle\phi_S\rangle}{M_F}\right)^2 \langle\phi_{WS}\rangle \tag{1.10}$$

via the exchange of two $\langle\phi_S\rangle$ tadpoles, in addition to the usual $\langle\phi_{WS}\rangle$ tadpole, through two appropriately charged vector-like fermion intermediate states [1]. We identify $\epsilon_f = \langle\phi_S\rangle/M_F$ as the $U(1)_f$ flavour symmetry breaking parameter. In general we expect mass matrix elements of the form:

$$M(i,j) = \gamma_{ij}\epsilon_f^{n_{ij}}\langle\phi_{WS}\rangle, \quad \gamma_{ij} = \mathcal{O}(1), \quad n_{ij} = \mid Q_f(\psi_{L_i}) - Q_f(\psi_{R_j}) \mid \tag{1.11}$$

between the left- and right-handed fermion components. So the *effective* SM Yukawa couplings of the quarks and leptons to the Weinberg-Salam Higgs field $y_{ij} = \gamma_{ij}\epsilon_f^{n_{ij}}$ can consequently be small even though all *fundamental* Yukawa couplings of the "true" underlying theory are of $\mathcal{O}(1)$. However it appears [17] not possible to explain the fermion mass spectrum with an anomaly free set of flavour charges in an $SMG \times U(1)_f$ model. It is necessary to introduce SMG-singlet fermions with non-zero $U(1)_f$ charge to cancel the $U(1)_f^3$ gauge anomaly (as in $MSSM \times U(1)_f$ models which also use anomaly cancellation via the Green-Schwarz mechanism [18]) or by extending the SM gauge group further (as in the AGUT model [19] based on the $SMG^3 \times U(1)_f$ gauge group).

6 Neutrino Mass and Mixing Problem

Physics beyond the SM can generate an effective light neutrino mass term

$$\mathcal{L}_{\nu-mass} = \sum_{i,j} \psi_{i\alpha}\psi_{j\beta}\epsilon^{\alpha\beta}(M_\nu)_{ij} \tag{1.12}$$

in the Lagrangian, where $\psi_{i,j}$ are the Weyl spinors of flavour i and j, and $\alpha, \beta = 1, 2$. Fermi-Dirac statistics mean that the mass matrix M_ν must be symmetric. In models with chiral flavour symmetry we typically expect the elements of the mass matrices to have different orders of magnitude. The charged lepton matrix is then expected to give only a small contribution to the lepton mixing. As a result of the symmetry of the neutrino mass matrix and the hierarchy of the mass matrix elements it is natural to have an almost degenerate pair of neutrinos, with nearly maximal mixing[20]. This occurs when an off-diagonal element dominates the mass matrix.

The recent Super-Kamiokande data on the atmospheric neutrino anomaly strongly suggests large $\nu_\mu - \nu_\tau$ mixing with a mass squared difference of order $\Delta m^2_{\nu_\mu\nu_\tau} \sim 10^{-3}$ eV^2. Large $\nu_\mu - \nu_\tau$ mixing is given by the mass matrix

$$M_\nu = \begin{pmatrix} \times & \times & \times \\ \times & \times & A \\ \times & A & \times \end{pmatrix} \tag{1.13}$$

where $\times$ denotes small elements and we have $\Delta m^2_{23} \ll \Delta m^2_{12} \sim \Delta m^2_{13}$, $\sin^2\theta_{23} \sim 1$ However, this hierarchy in Δm^2's is inconsistent with the small angle (MSW) solution to the solar neutrino problem, which requires $\Delta m^2_{12} \sim 10^{-5}$ eV^2. Also the theoretically attractive solution [21] of the atmospheric and solar neutrino problems, using maximal $\nu_e - \nu_\mu$ mixing, seems to be ruled out by the zenith angular distribution of the Super-Kamiokande data.

Hence we need extra structure for the mass matrix such as having several elements of the same order of magnitude. For example:

$$M_\nu = \begin{pmatrix} a & A & B \\ A & \times & \times \\ B & \times & \times \end{pmatrix} \tag{1.14}$$

with $A \sim B \gg a$. This gives

$$\frac{\Delta m^2_{12}}{\Delta m^2_{23}} \sim \frac{a}{\sqrt{A^2+B^2}}. \tag{1.15}$$

The mixing is between all three flavours and is given by the mixing matrix

$$U_\nu \sim \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}}\cos\theta & \frac{1}{\sqrt{2}}\cos\theta & -\sin\theta \\ \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{\sqrt{2}}\sin\theta & \cos\theta \end{pmatrix} \tag{1.16}$$

where $\theta = \tan^{-1}\frac{B}{A}$. So we have large $\nu_\mu - \nu_\tau$ mixing with $\Delta m^2 = \Delta m^2_{23}$, and nearly maximal electron neutrino mixing with $\Delta m^2 = \Delta m^2_{12}$. The atmospheric neutrino anomaly requires $\sin^2 2\theta \gtrsim 0.7$ or $1/2 \lesssim B/A \lesssim 2$. The solar neutrino problem is explained by vacuum oscillations, although whether it is an 'energy independent' or a 'just-so' solution depends on the value of the mass squared difference ratio in eq. (1.15).

There is also some difficulty in obtaining the required mass scale for the neutrinos. In models such as the AGUT the neutrino masses are generated via super-heavy intermediate fermions in a see-saw type mechanism. This leads to too small neutrino masses:

$$m_\nu \lesssim \frac{\langle\phi_{WS}\rangle^2}{M_F} \sim 10^{-5} \text{ eV}, \tag{1.17}$$

for $M_F = M_{Planck}$ (in general m_ν is also supressed by the chiral charges). So we need to introduce a new mass scale into the theory. Either some intermediate particles with mass $M_F \lesssim 10^{15}$ GeV, or an $SU(2)$ triplet Higgs field Δ with $\langle\Delta^0\rangle \sim 1$ eV is required. Without further motivation the introduction of such particles is *ad hoc*.

7 Acknowledgement

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Extending the Standard Model by Includind Right-Handed Neutrinos

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Abstract

The structure of the neutrino mass matrices is investigated, in a scheme where the minimal three-family Standard Model is extended by including right-handed neutrinos. No assumption is made about the presence of a large mass scale, like in the see-saw scheme. By demanding that the neutrino mass matrices have a specific form with a "Majorana democratic texture", Majorana mass spectra with three massless (light) neutrinos and either two or three massive neutrinos, are obtained.

1 The Standard Model with two right-handed neutrinos

In the minimal Standard Model it is assumed that the neutrinos have no mass, and no right-handed neutrinos are included in the model. Although there is still no conclusive experimental evidence for massive neutrinos, there is no good physical reason for excluding right-handed neutrinos. To introduce right-handed neutrinos is actually the simplest way of extending the Standard Model. Whereas for example the addition of a fourth standard family does not add any new features to the model, the introduction of right-handed neutrinos adds structures such as massive neutrinos, which could be the answer to the solar neutrino deficit and the atmospheric neutrino puzzle. Massive neutrinos are also prime candidates for hot dark matter. In such a scheme, we may also get lepton mixing and CP-violation in the leptonic sector.

In the Standard Model adding right-handed neutrinos results in a generic neutrino mass matrix of the form

$$\mathcal{M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \tag{2.1}$$

Unless the Higgs sector is modified, m_L is zero. If the lepton number is to be conserved, $m_L = m_R = 0$, and the neutrino is a four-component Dirac spinor endowed with mass by the standard Higgs mechanism with one Higgs doublet. If however lepton number conservation is not imposed, nonvanishing Majorana mass terms from m_L and/or m_R are allowed.

In the case with one left-handed and one right-handed neutrino and with $m_L = 0$, $\mathcal{M}$ corresponds to two nonvanishing mass eigenvalues. With the the assumption $m_D \ll m_R$, one obtains one very light and one very heavy mass value, m_D^2/m_R and m_R correspondingly. This is the "standard" see-saw mechanism for generating light neutrino masses.

We investigate an alternative scheme, namely the possibility of obtaining very light neutrino masses by including right-handed neutrinos, but without making any mass scale assumptions.

The simplest case, with three left-handed but only one right-handed neutrino included, gives rise to two very light and two massive neutrino states [1]. We however want a situation with three light neutrinos. Therefore, we consider the case with two right-handed neutrinos added to the minimal standard model with three families and one Higgs doublet. In the mass basis of the charged lepton sector the most general form of the neutrino mass term is

$$\mathcal{L}_{(\nu-mass)} = -\frac{1}{2}\bar{N}\mathcal{M}N^C + h.c. \tag{2.2}$$

where N contains the neutrino fields, and $\mathcal{M}$ is the neutrino mass matrix

$$\mathcal{M} = \begin{pmatrix} \mathbf{0} & \mathbf{A} \\ \tilde{\mathbf{A}} & \mathbf{M} \end{pmatrix} \tag{2.3}$$

Here the (Dirac) matrix $\mathbf{A}$ comes from the non-diagonal interactions of the left-handed and the right-handed neutrinos with the Higgs doublet, $\tilde{\mathbf{A}}$ is the transpose of $\mathbf{A}$, and the (Majorana) matrix $\mathbf{M}$ corresponds to the self-couplings of the right-handed neutrinos. As they are singlets they do not need the Higgs to acquire mass.

In the case of two right-handed neutrinos,

$$N = \mathrm{col}(\nu''_{1L}, \nu''_{2L}, \nu''_{3L}, \nu'^C_{1R}, \nu'^C_{2R})$$

and

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & 0 & a_1 & b_1 \\ 0 & 0 & 0 & a_2 & b_2 \\ 0 & 0 & 0 & a_3 & b_3 \\ a_1 & a_2 & a_3 & M_1 & 0 \\ b_1 & b_2 & b_3 & 0 & M_2 \end{pmatrix} \tag{2.4}$$

We want three zero and two nonvanishing mass eigenvalues. The characteristic equation reads

$$\lambda[\lambda^4 - \lambda^3 trM + \lambda^2[detM - tr(\tilde{A}A)] + \lambda tr(\hat{M}\tilde{A}A) + det(\tilde{A}A)] = 0, \tag{2.5}$$

so there is already one "automatically" vanishing mass eigenvalue, only two conditions need therefore to be satisfied in order to have three vanishing neutrino mass eigenvalues and a nonvanishing Majorana sector (i.e. $detM \neq 0$), viz.

$$det(\tilde{A}A) = 0 \quad \text{and} \quad tr(\hat{M}\tilde{A}A) = 0 \tag{2.6}$$

where

$$\hat{\mathbf{M}} = \begin{pmatrix} M_2 & 0 \\ 0 & M_1 \end{pmatrix} \tag{2.7}$$

This can be expressed as the conditions

$$\bar{a} = x\bar{c}, \quad \bar{b} = y\bar{c}, \quad \text{and} \quad M_1 y^2 + M_2 x^2 = 0 \tag{2.8}$$

where x, y are real numbers, $\bar{a} = (a_1, a_2, a_3)$, $\bar{b} = (b_1, b_2, b_3)$ and $\bar{c}$ is a unit 3-vector.

The states with vanishing masses at tree level are expected to acquire small radiative masses. These radiative corrections are due to the two massive states $\lambda_{\pm}$, and generated by one-loop diagrams with contributions from both Z and the neutral physical Higgs, as well as from two-W and two-Z exchange diagrams, since in this model there are flavour changing neutral currents.

The mass matrix $\mathcal{M}$ is diagonalized by means of a unitary 5x5-matrix $\mathbf{U}$, which may be parametrized in terms of four angles, $(\psi, \ \phi, \ \theta, \ \gamma)$, such that $\bar{c} = (-\cos\psi, \sin\phi\sin\psi, \cos\phi\sin\psi)$, and

$$\frac{x}{y} = \tan\theta, \quad \text{whereby} \quad M_1 = -\tan^2\theta M_2 \tag{2.9}$$

In terms of these angles, the non-vanishing mass eigenvalues and the mixing matrix are

$$\begin{aligned} \lambda_+ &= M_1 \frac{(1-\tan^2\theta)}{\tan^2\theta} \frac{\tan^2\gamma}{(1-\tan^2\gamma)} \\ \lambda_- &= M_1 \frac{(1-\tan^2\theta)}{\tan^2\theta} \frac{1}{(1-\tan^2\gamma)} \end{aligned} \tag{2.10}$$

and

$$\mathbf{U} = \frac{\mathbf{\Upsilon}}{s_{2\gamma}c_{2\theta}} \begin{pmatrix} s_{2\gamma}c_{2\theta}s_\psi & s_{2\gamma}c_{2\theta}s_\phi c_\psi & s_{2\gamma}c_{2\theta}c_\phi c_\psi & 0 & 0 \\ 0 & s_{2\gamma}c_{2\theta}c_\phi & -s_{2\gamma}c_{2\theta}s_\phi & 0 & 0 \\ -c_{2\gamma}s_{2\theta}c_\psi & c_{2\gamma}s_{2\theta}s_\phi s_\psi & c_{2\gamma}s_{2\theta}c_\phi s_\psi & c_\theta S & -s_\theta S \\ -c_\gamma c_\psi S & c_\gamma s_\phi s_\psi S & c_\gamma c_\phi s_\psi S & -s_\theta c_\gamma(c_{2\theta}+c_{2\gamma}) & -c_\theta c_\gamma(c_{2\theta}-c_{2\gamma}) \\ -s_\gamma c_\psi S & s_\gamma s_\phi s_\psi S & s_\gamma c_\phi s_\psi S & s_\theta s_\gamma(c_{2\theta}-c_{2\gamma}) & c_\theta s_\gamma(c_{2\gamma}+c_{2\theta}) \end{pmatrix}, \tag{2.11}$$

correspondingly. Here $S = \sqrt{c_{2\theta}^2 - c_{2\gamma}^2}$, and $\mathbf{\Upsilon}$ is a phase matrix which was introduced in order to make all mass eigenvalues positive; a reminder of the fact that the neutrino and the antineutrino have opposite CP parities. With this mixing matrix, the neutral current term takes the form

$$\mathcal{L}_{NC} = \frac{g}{2\cos\theta_W}(\bar{\nu}_{1L}, \bar{\nu}_{2L}, \bar{\nu}_{3L}, \bar{\nu}_{+L}, \bar{\nu}_{-L})\gamma^\lambda \mathbf{\Omega} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \\ \nu_{+L} \\ \nu_{-L} \end{pmatrix} Z_\lambda \tag{2.12}$$

where $\mathbf{\Omega}$ is the matrix $\mathbf{\Omega} = \mathbf{U}diag(1,1,1,0,0)\mathbf{U}^\dagger$, which is but a unitary transformation of the matrix $diag(1,1,1,0,0)$. The trace of $\mathbf{\Omega}$ is therefore $tr(\mathbf{\Omega}) = 3$, and from $\mathbf{\Omega}\mathbf{\Omega}^\dagger = \mathbf{\Omega}^2 = \mathbf{\Omega}$, we see that $tr(\mathbf{\Omega}\mathbf{\Omega}^\dagger) = tr(\mathbf{\Omega}) = 3$. The neutral current coupling coefficients thus satisfy

$$\sum_{j,k=1}^{5} |\mathbf{\Omega}_{jk}|^2 = 3, \tag{2.13}$$

where the right-hand side is just the number of left-handed leptonic doublets, i.e the number of families.

The invisible width of the Z is determined from studies of Z-production in e^+e^- collisions [2], by subtracting the measured visible partial widths, corresponding to Z decays into quarks and leptons, from the total Z width. In our scenario the invisible width of the Z's is always smaller than predicted by the minimal Standard Model. The reason for this is that in a model with n left-handed lepton doublets and $k-n$ right-handed neutrinos, the effective number $< N_\nu >$ of neutrinos is defined by the invisible Z width, i.e.

$$\Gamma(\mathrm{Z} \to \nu'\mathrm{s}) = \Gamma_0 < \mathrm{N}_\nu >= \Gamma_0 \sum_{\mathrm{i,j=1}}^{\mathrm{k}} \mathrm{X_{ij}}|\Omega_{\mathrm{ij}}|^2 \tag{2.14}$$

where Γ_0 is the standard width for a massless neutrino pair and the X_{ij} are the phase space and matrix element suppression factors due to the nonvanishing neutrino masses. Now, as the $X'_{ij}s$ are bounded by unity, $\Gamma(\mathrm{Z} \to \bar{\nu}\nu) \leq \mathrm{n}\Gamma_0$, and in our case, $n = 3$. This means that in a scheme with right-handed neutrinos, no definite conclusion can be drawn from neutrino-counting at the Z-peak.

In neutrinoless double-beta $(\beta\beta)_{0\nu}$ decay, when the neutrino masses are very small, we can define the effective neutrino mass $< m_\nu >$ as

$$< m_\nu >= |\sum_j m_{\nu_j} U^2_{\nu_j e}|, \tag{2.15}$$

In our model, $< m_\nu >= 0$, and the current experimental limit [3] from the research for $(\beta\beta)_{0\nu}$ decay is $| < m_\nu > | \lesssim 1 - 2eV$. Similar results hold for double-muon and double-tau decays.

2 The Standard Model with three right-handed neutrinos

The case of three right-handed neutrinos added to the Standard Model is analogous to the case with two right-handed neutrinos. Like in the case with two neutrinos, the mass matrix has the form (3.6), but now the matrices $\mathbf{M}$ and $\mathbf{A}$ are 3x3. In order to get three non-vanishing and three vanishing neutrino masses (at tree level), it is necessary that $det(\tilde{A}A) = 0$, $tr(M\hat{A}\tilde{\hat{A}}) = 0$ and $tr(\hat{A}\tilde{\hat{A}}) - tr(\hat{M}\tilde{A}A) = 0$, where

$$\hat{A}_j = \frac{1}{2}\epsilon_{jkl}\epsilon_{\beta\gamma}A_{k\beta}A_{l\gamma}$$
$$\hat{M}_j = \frac{1}{2}\epsilon_{jkl}\epsilon_{\beta\gamma}M_{k\beta}M_{l\gamma},$$

In analogy with (2.8), this can be obtained by demanding that

$$\begin{aligned}&(a_1,a_2,a_3) = x\bar{c}, \quad (b_1,b_2,b_3) = y\bar{c}, \quad (d_1,d_2,d_3) = z\bar{c}\\ &\text{and} \quad M_2M_3x^2 + M_1M_3y^2 + M_1M_2z^2 = 0\end{aligned} \tag{2.16}$$

where x, y, z are real non-zero numbers and $\bar{c}$ is a unit vector. $\mathbf{A}$ can then be written as

$$\mathbf{A} = \mathbf{CNX} = \begin{pmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \\ 0 & 0 & c_3 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} \tag{2.17}$$

The mass matrix can now be written in a form that displays the *Majorana democratic texture*

$$\mathcal{M} = \begin{pmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{X} \end{pmatrix}\begin{pmatrix} \mathbf{0} & \mathbf{N} \\ \mathbf{N} & \mathbf{m} \end{pmatrix}\begin{pmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{X} \end{pmatrix}, \tag{2.18}$$

where $\mathbf{0}$ is a 3x3 matrix where all the matrix elements are zero, and $\mathbf{N}$, $\mathbf{C}$ and $\mathbf{X}$ are defined by (2.17), and

$$\mathbf{m} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

where $m_1 = M_1/x^2$, $m_2 = M_2/y^2$, $m_3 = M_3/z^2$ satisfy $m_1m_2 + m_1m_3 + m_2m_3 = 0$.
The neutrino mass matrix (2.18) is a general Dirac-Majorana mass matrix. Its form gives rise to three light and three massive neutrinos, in a scheme without any assumption about the presence of a large mass scale. It is tempting to speculate that the democratic texture displayed by this matrix tells us something about the structure of the mass matrices of the charged fermion sector. The ansatz that naturally occurs, is a charged fermion mass matrix of the form

$$m = X\mathbf{N}Y + \Lambda \tag{2.19}$$

where X and Y are (diagonal) 3x3 matrices, and Λ is a matrix such that $\Lambda_{ij} \ll X_iY_j$.

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Multiple Point Principle and Phase Transition in Gauge Theories

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Standard model unifying QCD with Glashow–Salam–Weinberg electroweak theory well describes all experimental results known today. Most efforts to explain the Standard model are devoted to Grand unification theory (GUT). The precision of the LEP–data allows to extrapolate three running constants $\alpha_i(\mu)$ of the Standard model ($i = 1, 2, 3$ corresponds to U(1), SU(2), SU(3) groups) to high energies with small errors and we are able to perform consistency checks of GUTs.

In the Standard model based on the group

$$SMG = SU(3)_c \otimes SU(2)_L \otimes U(1) \tag{3.1}$$

the usual definitions of the coupling constants are used:

$$\alpha_1 = \frac{5}{3}\frac{\alpha}{\cos^2\theta_{\overline{MS}}}, \quad \alpha_2 = \frac{\alpha}{\sin^2\theta_{\overline{MS}}}, \quad \alpha_3 \equiv \alpha_S = \frac{g_S^2}{4\pi}, \tag{3.2}$$

where α and α_s are the electromagnetic and strong fine structure constants, respectively. All of these couplings, as well as the weak angle, are defined here in the Modified minimal substraction scheme ($\overline{MS}$). Using experimentally given parameters and the renormalization group equations, it is possible to extrapolate the experimental values of three inverse running constants $\alpha_i^{-1}(\mu)$ to the Planck scale: $\mu_{Pl} = 1.22 \cdot 10^{19}$GeV.

The comparison of the evolutions of the inverses of the running coupling constants to high energies in the Minimal Standard model (MSM) (with one Higgs doublet) and in the Minimal Supersymmetric Standard model (MSSM) (with two Higgs doublets) gives rise to the existence of the grand unification point at $\mu_{GUT} \sim 10^{16}$GeV only in the case of MSSM (see Ref.[1]). This observation is true for a whole class of GUT's that break to the Standard model group in one step, and which predict a "grand desert" between the weak (low) and the grand unification (high) scales. If grand desert indeed exists, and the supersymmetry is established at future colliders then we shall eventually be able to use the coupling constant unification to probe the new physics near the unification and Planck scales.

Scenarios based on the Anti-grand unification theory ($AGUT$) was developed in Refs.[2]-[10] as an alternative to GUT's. The assumption of AGUT is: the supersymmetry doesn't exist up to the Planck scale. There is no new physics (new particles, superpartners) up to this scale. This means that the renormalization group extrapolation of experimentally determined couplings to the Planck scale is contingent not encountering new particles.

AGUT suggests that at the Planck scale μ_{Pl}, considered as a fundamental scale, there exists the more fundamental gauge group G, containing N_{gen} copies of the Standard model group SMG:

$$G = SMG_1 \otimes SMG_2 \otimes \ldots \otimes SMG_{N_{gen}} \equiv (SMG)^{N_{gen}}, \tag{3.3}$$

where the integer N_{gen} designates the number of quark and lepton generations .

SMG by definition is the following factor group:

$$SMG = S(U(2) \times U(3)) = \frac{U(1) \times SU(2) \times SU(3)}{\{(2\pi, -1^{2\times 2}, e^{i2\pi/3}1^{3\times 3})^n | n \in Z\}}. \tag{3.4}$$

If $N_{gen} = 3$, then the fundamental gauge group G is:

$$G = (SMG)^3 = SMG_1 \otimes SMG_2 \otimes SMG_3, \tag{3.5}$$

or the generalized G:

$$G = (SMG)^3 \otimes U(1)_f \tag{3.6}$$

which follows from the fitting of fermion masses (see Ref.[11]).

The group $G = (SMG)^3 \otimes U(1)_f$ is a maximal gauge transforming (nontrivially) the 45 Weyl fermions of the Standard model (which it extends) without unifying any of the irreducible representations of the group of the latter.

A base of the AGUT is the Multiple Point Principle (MPP) proposed several years ago by D.L.Bennett and H.B.Nielsen [7]-[8]. Another name for the same principle is the "maximally degenerate vacuum principle" (MDVP).

According to this Principle, Nature seeks a special point – the multiple critical point (MCP) where the group G undergoes spontaneous breakdown to the diagonal subgroup:

$$G \to G_{diag.subgr.} = \{g, g, g \parallel g \in SMG\} \tag{3.7}$$

which is identified with the usual (lowenergy) group SMG.

The idea of the MPP has its origin from the lattice investigations of gauge theories. In particular, Monte Carlo simulations on lattice of U(1)–, SU(2)– and SU(3)– gauge theories indicate the existence of a triple critical point. Using theoretical corrections to the Monte Carlo results on lattice, it is possible to make slightly more accurate predictions of AGUT for the Standard model fine-structure constants.

MPP assumes that SM gauge couplings do not unify and predicts their values at the Planck scale in terms of critical couplings taken from the lattice gauge theory:

$$\alpha_i(M_{Pl}) = \frac{\alpha_i^{crit}}{N_{gen}} = \frac{\alpha_i^{crit}}{3} \tag{3.8}$$

for $i = 2, 3$ and

$$\alpha_1(M_{Pl}) = \frac{\alpha_1^{crit}}{\frac{1}{2}N_{gen}(N_{gen}+1)} == \frac{\alpha_1^{crit}}{6} \tag{3.9}$$

for U(1).

This means that at the Planck scale the fine structure constants $\alpha_Y \equiv \frac{3}{5}\alpha_1$, α_2 and α_3, as chosen by Nature, are just the ones corresponding to the multiple critical point (MCP) which is a point where all action parameter (coupling) values meet in the phase diagram of the regularized Yang-Mills $(SMG)^3$ - gauge theory. Nature chooses coupling constant values such that a number of vacuum states have the same energy density - degenerate vacua. Then all (or at least maximum) number of phases convene at the Multiple Critical Point and the different vacua are degenerate.

The extrapolation of the experimental values of the inverses $\alpha^{-1}_{Y,2,3}(\mu)$ to the Planck scale μ_{Pl} by the renormalization group formulas (under the assumption of a "desert" in doing the extrapolation with one Higgs doublet) gives us the following result:

$$\alpha_Y^{-1}(\mu_{Pl}) = 55.5; \quad \alpha_2^{-1}(\mu_{Pl}) = 49.5; \quad \alpha_3^{-1}(\mu_{Pl}) = 54. \tag{3.10}$$

Using Monte Carlo results on lattice, AGUT predicts (Refs.[6]-[8]):

Table 1

Group	AGUT predictions	"Experiment" – the extrapolation of the SM results to the Planck scale
$U(1)$	$\alpha_Y^{-1}(\mu_{MCP}) = 55 \pm 6$	$\alpha_Y^{-1}(\mu_{Pl}) \approx 55.5$
$SU(2)$	$\alpha_2^{-1}(\mu_{MCP}) = 49.5 \pm 3$	$\alpha_2^{-1}(\mu_{Pl}) \approx 49.5$
$SU(3)$	$\alpha_3^{-1}(\mu_{MCP}) = 57 \pm 3$	$\alpha_3^{-1}(\mu_{Pl}) \approx 54$

For $U(1)$ - gauge lattice theory the authors of Ref.[12] have investigated the behaviour of the effective fine structure constant in the vicinity of the critical point and they have obtained:

$$\alpha_{crit} \approx 0.2. \tag{3.11}$$

We gave put forward the calculations of the fine structure constant in $U(1)$ - gauge theory, suggesting that the modification of the action form might not change too much the critical value of the effective coupling constant.

The phase transition between the confinement and "Coulomb" phases in the regularized U(1)-gauge theory was investigated in Ref.[13]. Instead of the lattice hypercubic regularization it was considered rather new regularization using Wilson loop (nonlocal) action :

$$S = \int\limits_0^\infty d\log(\frac{R}{a})\beta(R)R^{-4} \sum_{average} ReTrexp\left[i\oint_{C(R)} \hat{A}_\mu(x)dx^\mu\right] \tag{3.12}$$

in approximation of circular Wilson loops $C(R)$ of radii $R \geq a$. Here ($\sum\limits_{average}$) denotes the average over all positions and orientations of the Wilson loops $C(R)$ in 4-dimensional (Euclidean) space. It was shown:

$$\alpha_{crit} \approx 0.204, \tag{3.13}$$

in correspondence with Monte Carlo simulation result (3.11) on the lattice.

The further investigations confirm the "universality" of the critical coupling constants.

In lattice gauge theories monopoles are artifacts of the regularization. Let us consider a new assumption: <u>monopoles exist physically</u>. With aim to confirm the "universality" of the critical coupling constant in the regularized U(1)–gauge theory with matter we have investigated quantum electrodynamics with scalar Higgsed monopoles for a phase transition (Re.[14]).

Considering the Lagrangian which describes the interaction of the Higgsed scalar monopole field $\Phi(x)$ with dual gauge field $\hat{C}_\mu = gC_\mu$, we have:

$$L(x) = -\frac{1}{4g^2}\left(\hat{G}_{\mu\nu}\right)^2 + |D_\mu\Phi|^2 - U(\Phi), \tag{3.14}$$

where

$$G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu, \qquad D_\mu = \partial_\mu + i\hat{C}_\mu$$

and

$$U(\Phi) = -\mu^2|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4 \tag{3.15}$$

are the dual field strength, covariant derivative and Higgs potential, respectively.

The complex scalar field $\Phi = \frac{1}{\sqrt{2}}(\phi + i\chi)$ contains the Higgs and Goldstone boson fields $\phi(x)$ and $\chi(x)$, respectively.

The free energy $F[\Phi_B]$ may be expressed in terms of the functional integral (in Euclidean space) over the shifted action:

$$S = \int d^4x L^{(E)}(x) \tag{3.16}$$

with a shift:

$$\Phi(x) -> \ \Phi_B + \tilde{\Phi}(x) \tag{3.17}$$

where Φ_B is a background field. We can obtain the effective potential $V_{eff} = F[\Phi_B, g^2, \mu^2, \lambda]$ for a constant background field: $\Phi_B = \phi_B = const$. It was calculated in the one-loop approximation (see Refs.[15]):

$$V_{eff} = \frac{1}{2}\mu^2{\phi_B}^2 + \frac{1}{4}\lambda{\phi_B}^4 + \frac{3g^4}{64\pi^2}{\phi_B}^4\log\frac{{\phi_B}^2}{M^2}$$

$$+\frac{1}{64\pi^2}(\mu^2 + 3\lambda{\phi_B}^2)^2\log\frac{\mu^2 + 3\lambda{\phi_B}^2}{M^2} + \frac{1}{64\pi^2}(\mu^2 + \lambda{\phi_B}^2)^2\log\frac{\mu^2 + \lambda{\phi_B}^2}{M^2}. \tag{3.18}$$

This effective potential has several minima, the position of which is dependent of g^2, μ^2 and λ.

It is easy to see that the first local minimum occurs at $\phi_B = 0$ and corresponds to so-called "symmetric phase" ("Coulomb phase" in our description). The phase transition from "Coulomb phase" to the confinement phase occurs if the second local minimum at $\phi_B = \phi_0$ is degenerate with the first local minimum at $\phi = 0$. As a result, we have two equations:

$$V_{eff}(\phi_0) = 0, \tag{3.19}$$

$$V'_{eff} = \frac{dV_{eff}}{d\phi_B}|_{\phi_B=\phi_0} = 0. \tag{3.20}$$

The solution of these equations gives us the following relation:

$$g^4 + \frac{16\pi^2}{3}\lambda_{run} + \frac{10}{3}\lambda^2_{run} = 0, \tag{3.21}$$

where, according to Eq.(3.18):

$$\lambda_{run} = \lambda + \frac{1}{16\pi^2}(3g^4 \log\frac{\phi^2}{M^2} + 9\lambda^2 \log\frac{\mu^2 + 3\lambda\phi^2}{M^2} + \lambda^2 \log\frac{\mu^2 + \lambda\phi^2}{M^2}). \tag{3.22}$$

The third requirement $V''_{eff} \geq 0$, which means the existence of minimum of V_{eff} at the $\phi = \phi_0$, leads to the following equation at the border of two phases:

$$V''_{eff}|_{\phi_B=\phi_0} = 0. \tag{3.23}$$

The solution of this equation, together with Eq.(3.21), gives us the following results:

$$\lambda_{run} = -\frac{4}{5}\pi^2,$$

$$g^4{}_{crit} = \frac{32}{15}\pi^4. \tag{3.24}$$

Using Dirac relations:

$$eg = 2\pi, \qquad \alpha_e\alpha_m = \frac{1}{4}, \tag{3.25}$$

where e is the electric charge, $\alpha_e \equiv \alpha = e^2/4\pi$ and $\alpha_m = g^2/4\pi$ is the electric and magnetic fine structure constants, respectively, we can easily obtain (with help of the result (3.24)):

$$\alpha_{crit} \approx 0.22. \tag{3.26}$$

This value of the critical fine structure constant is in correspondence with the results of Monte Carlo simulations on lattice ($\alpha_{crit} \approx 0.20$ [12]) and the regularized gauge theory using nonlocal Wilson loop action: $\alpha_{crit} \approx 0.204$ [13].

Thus, an idea of the "universality" (regularization independence) of the critical couplings was confirmed in U(1)– gauge theories. Such a (maybe approximate) "universality" of the critical coupling constants is needed for the fine structure constant predictions claimed from AGUT [6]-[8].

We were interested also in a question: is it possible to include gravity for a phase transition at the Planck scale? The main idea of our new work [16] is to include the gravity, considering the simplest action for scalar or fermion monopoles which interact with dual gauge fields in presence of the gravity. In the case of the fermion monopoles we have the following action:

$$S = \int d^4x\sqrt{-\tilde{g}}[\frac{1}{16\pi G}R + \frac{1}{4g^2}(\hat{G}_{\mu\nu})^2 + \bar{\psi}(ig^{\mu\nu}\gamma_\mu D_\nu - M)\psi]. \tag{3.27}$$

Here $g^{\mu\nu}$ is a metric tensor, $\tilde{g}$ is its determinant and D_μ is a covariant derivative, containing also the connection. G is the gravitational constant and R is the scalar curvature. But now $g^{\mu\nu}$ plays a role of the field variable. The variation of the action (3.27) over $g^{\mu\nu}, \psi$ and C_μ gives us three equations of motion: 1) the equation for $g^{\mu\nu}$; 2) the second one for ψ and 3) the third one for C_μ. Using the first equation, we can consider the second order formalism, excluding the gravitational field. This procedure leads to the appearance of the 4-fermion term in the resulting effective action. Such a term has the coupling constant related with the gravitational constant G and is responsible for the phenomenon similar to the formation of Cooper pairs in superconductivity. Why is it possible?

Let us assume that monopoles have a large mass $M < M_{Pl}$, but comparable with the Planck mass M_{Pl}. At $\mu \geq M$ the running (electric) fine structure constant $\alpha(\mu)$ would be also renormalized by monopole loops and increase rapidly. Then, according to the relations (3.25), $\alpha_m = \frac{1}{4\alpha}$ decreases rapidly and at some

$$\alpha_m^{crit} = (M/M_{Pl})^2 \tag{3.28}$$

the repulsion of two monopoles (with the same magnetic charges g) becomes equal to the gravitational attraction between them. But for $\alpha_m < \alpha_m^{crit}$ the gravitational attraction will be larger than electromagnetic repulsion of monopoles and the formation of scalar bound states (with magnetic charge $2g$) is quite possible. Their condensate is analogous to the Cooper pairs in superconductivity and leads to the formation of the vortices (strings). In such a theory the thikness of "strings" plays role of the regularization parameter.

Maybe this is a way to construct supersymmetric strings and to consider the violation of the supersymmetry at the Planck scale.

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Unification of Spins and Charges Enabels Unification of All Interactions

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Abstract

In a space of d Grassmann (anticommuting) coordinates two types of generators of Lorentz transformations, one of spinorial and the other of vectorial character, define the representations of the group $SO(1,13)$ and of the subgroups $SO(1,3), SU(3), SU(2), U(1)$, for fermions and bosons, respectively, unifying all the internal degrees of freedom - spins and Yang-Mills charges. When accordingly all the interactions are unified, Yang-Mills fields appear as a part of a gravitational field. The theory suggests that elementary particles are either in the fermionic representations with respect to the groups determining the spin and the charges, or they are in the bosonic representations with respect to the groups, which determine the spin and the charges. It also suggests four rather than three generations of quarks and leptons and says that the left handed weak charge doublets are in the same multiplet as the right handed weak charge singlets.

1 Introduction

Since Newton, the understanding of the laws of Nature has developed from the laws leaving (almost) infinitely many parameters free (all possible masses as well as forces) to be determined by the experiment, to the unified quantum theory of electromagnetic, weak and colour interaction, which only has around 20 parameters.

What today is accepted as elementary particles and fields are either fermionic fields with all the charges in the fundamental representations with respect to the groups $U(1), SU(2), SU(3)$ or bosonic fields with all the charges in the adjoint representations with respect to the same groups. There exists no known fermion (yet) with charges in the adjoint representations and no known boson yet with charges in the fundamental representations. The not yet observed Higgs scalar, however, which appears in the Standard model as a weak charge doublet, seems to be of such a "mixed" type (or it might be a constituent particle[1]).

Not only has the Standard electroweak model free parameters, it also has several assumptions, which have no theoretical support (not in the Standard model concept): i) There are three families of quarks and leptons, which are massless and which gain masses through the Yukawa couplings with the Higgs field. ii) Quarks carry colour ($SU(3)$) charges. If left handed they carry weak ($SU(2)$) and $Y(U(1))$ charges, if right handed they carry only Y charges. The left handed leptons carry weak and Y charges, right handed ones carry either only Y charges, or no charge at all. iii) The Y charges are chosen in a way to reproduce the electromagnetic charges of physical particles. iv) Charges of fermions are described by either the fundamental representations of the groups $SU(3), SU(2)$ and $U(1)$, or they are singlets with respect to some or all of these groups, charges of bosons are described by the adjoint representations of these groups, or they are singlets with respect to some or all of these groups. v) There exists a (complex) scalar field with respect to the group $SO(1,3)$, which is the colour singlet and the weak doublet and carry the Y charge.

The Standard model does not say: i) Why $SO(1,3) \times SU(3) \times SU(2) \times U(1)$ are the imput symmetries of the model? ii) Why fermions are left handed weak doublets and right handed weak singlets? iii) Where do the generations of fermions come from? Why there exist only three generations? iv) Where do the Yukawa couplings come from? Where does the Higgs come from?

In the Standard model the Y charges are free parameters of the model. Embedding the charge groups into $SU(5)$ fixes the Y charge uniquely, but leaves the connection between the handedness (that is spin) and charges undetermined. **To connect spins and charges, spins and charges should unify.**

In this talk I am proposing the approach which unifies spins and charges requiring that elementary fermions are in the spinorial representations with respect to all gauge groups and elementary bosons are in the vectorial representations with respect to all gauge groups. It also suggests that fermions, which are left handed weak charge doublets appear together with right handed weak charge singlets in the same multiplet, offering a way of understanding why left handed weak charge doublets and right handed weak charge singlets appear in the Standard electroweak model. The Yukawa couplings appear as a part of spin connections, which also define all gauge fields.

[1] We show that Higgs scalar is described by one of the bosonic representations of the proposed approach.

The space in the approach of mine has d commuting (ordinary) and d anticommuting (Grassmann) coordinates. **All the internal degrees of freedom, spins and charges, are described by the generators of the Lorentz transformations** in Grassmann space. All gauge fields - gravitational as well as Yang-Mills - are defined by supervielbeins.

In Grassmann space there are two types of generators of Lorentz transformations and translations: one is of spinorial character and determines properties of fermions, the other is of vectorial character and determines properties of bosons. Both types of generators are linear differential operators in Grassmann space. Their representations can be expressed as monomials of Grassmann coordinates θ^a. If $d \geq 14$ the generators of the subgroup $SO(1,3)$ of the group $SO(1,13)$ determine spins of fields, while generators of the subgroups $SU(3), SU(2), U(1)$ determine their charges.

The Lagrange function describing a particle on a supergeodesics, leads to the momentum of the particle in Grassmann space which is proportional to the Grassmann coordinate. This brings the Clifford algebra and the spinorial degrees of freedom into the theory. The supervielbeins, transforming the geodesics from the freely falling to the external coordinate system, depend on ordinary and Grassmann coordinates (the later determine spins and charges of fields) and carry accordingly the bosonic and the fermionic degrees of freedom, if expressed in terms of monomials of a Grassmann even and odd character, respectively .

The Yang-Mills fields appear as the contribution of gravity through spin connections and not through vielbeins as in the Kaluza-Klein theories. Because of that and because the generators of the Lorentz transformations in Grassmann space rather than in ordinary space determine charges of fields, the Planck mass of charged particles as in Kaluza-Klein theories seems not to appear in this approach. The Yukawa couplings may be explained by having the origin in spin connections as well. In such a case, however, mass terms are of the order of a Planck mass. (More about this approach can be found in Refs.[1, 2].)

2 Coordinate Grassmann Space and Linear Operators

In this section we briefly repeat a few definitions concerning a d-dimensional Grassmann space, linear Grassmann space spanned over the coordinate space, linear operators defined in this space and the Lie algebra of generators of the Lorentz transformations [2, 3].

We define a d-dimensional Grassmann space of real anticommuting coordinates $\{\theta^a\}$, $a = 0, 1, 2, 3, 5, 6, ..., d$, satisfying the anticommutation relations $\theta^a\theta^b + \theta^b\theta^a := \{\theta^a, \theta^b\} = 0$, called the Grassmann algebra [2, 3]. The metric tensor $\eta_{ab} = diag(1, -1, -1, -1, ..., -1)$ lowers the indices of a vector $\{\theta^a\} = \{\theta^0, \theta^1, ..., \theta^d\}, \theta_a = \eta_{ab}\theta^b$. Linear transformation actions on vectors $(\alpha\theta^a + \beta x^a)$, $\quad (\alpha\acute{\theta}^a + \beta\acute{x}^a) = L^a{}_b(\alpha\theta^b + \beta x^b)$, which leave forms $(\alpha\theta^a + \beta x^a)(\alpha\theta^b + \beta x^b)\eta_{ab}$ invariant, are called the Lorentz transformations. Here $(\alpha\theta^a + \beta x^a)$ is a vector of d anticommuting components and d commuting $(x^a x^b - x^b x^a = 0)$ components, and α and β are two complex numbers. The requirement that forms $(\alpha\theta^a + \beta x^a)(\alpha\theta^b + \beta x^b)\eta_{ab}$ are scalars with respect to the above linear transformations, leads to the equations $L^a{}_c L^b{}_d \eta_{ab} = \eta_{cd}$.

A linear space spanned over a Grassmann coordinate space of d coordinates has the dimension 2^d. If monomials $\theta^{\alpha_1}\theta^{\alpha_2}....\theta^{\alpha_n}$ are taken as a set of basic vectors with $\alpha_i \neq \alpha_j$, half of the vectors have an even (those with an even n) and half of the vectors have an odd (those with an odd n) Grassmann character. Any vector in this space may be represented as a linear superposition of monomials

$$f(\theta) = \alpha_0 + \sum_{i=1}^{d} \alpha_{a_1 a_2 .. a_i} \theta^{a_1}\theta^{a_2}....\theta^{a_i}, \quad a_k < a_{k+1}, \tag{4.1}$$

where constants $\alpha_0, \alpha_{a_1 a_2 .. a_i}$ are complex numbers and the ascending order of coefficients $a_1 < a_2 < ... < a_i$ is assumed.

In Grassmann space the left derivatives have to be distinguished from the right derivatives, due to the anticommuting nature of the coordinates [2, 3]. We shall make use of left derivatives $\overrightarrow{\partial}^{\theta}{}_a := \frac{\overrightarrow{\partial}}{\partial\theta^a}, \quad \overrightarrow{\partial}^{\theta a} := \eta^{ab}\overrightarrow{\partial}^{\theta}{}_b$, on vectors of the linear space of monomials $f(\theta)$, defined as follows: $\overrightarrow{\partial}^{\theta}{}_a \, \theta^b f(\theta) = \delta^b{}_a f(\theta) - \theta^b \overrightarrow{\partial}^{\theta}{}_a \, f(\theta)$.

Here α is a constant of either commuting $(\alpha\theta^a - \theta^a\alpha = 0)$ or anticommuting $(\alpha\theta^a + \theta^a\alpha = 0)$ character, and $n_{a\partial}$ is defined as follows $n_{AB} = \left\{ \begin{array}{ll} +1, & if\ A\ and\ B\ have\ Grassmann\ odd\ character \\ 0, & otherwise \end{array} \right\}$.

We define the following linear operators [1, 2]

$$p^{\theta}{}_a := -i\overrightarrow{\partial}^{\theta}{}_a, \quad \tilde{a}^a := i(p^{\theta a} - i\theta^a), \quad \tilde{\tilde{a}}^a := -(p^{\theta a} + i\theta^a). \tag{4.2}$$

According to the inner product defined in what follows, the operators $\tilde{a}^a$ and $\tilde{\tilde{a}}^a$ are either hermitian or antihermitian operators.

We define the generalized commutation relations (which follow from the corresponding Poisson brackets [1, 2]):

$$\{A, B\} := AB - (-1)^{n_{AB}} BA, \tag{4.3}$$

fulfilling the equation $\{A, B\} = (-1)^{n_{AB}+1}\{B, A\}$,

We find

$$\{p^{\theta a}, p^{\theta b}\} = 0 = \{\theta^a, \theta^b\}, \;\; \{p^{\theta a}, \theta^b\} = -i\eta^{ab}, \;\; \{\tilde{a}^a, \tilde{a}^b\} = 2\eta^{ab} = \{\tilde{\tilde{a}}^a, \tilde{\tilde{a}}^b\}, \;\; \{\tilde{a}^a, \tilde{\tilde{a}}^b\} = 0. \tag{4.4}$$

We see that θ^a and $p^{\theta a}$ form a Grassmann odd Heisenberg algebra, while $\tilde{a}^a$ and $\tilde{\tilde{a}}^a$ form the Clifford algebra.

We define the projectors

$$P_\pm = \frac{1}{2}(1 \pm \sqrt{(-)^{\tilde{\Upsilon}\tilde{\tilde{\Upsilon}}}}\tilde{\Upsilon}\tilde{\tilde{\Upsilon}}), \quad (P_\pm)^2 = P_\pm, \tag{4.5}$$

where $\tilde{\Upsilon}$ and $\tilde{\tilde{\Upsilon}}$ are the two operators defined for any dimension d as follows $\tilde{\Upsilon} = i^\alpha \prod_{a=0,1,2,3,5,..,d} \tilde{a}^a \sqrt{\eta^{aa}}$, $\;\tilde{\tilde{\Upsilon}} = i^\alpha \prod_{a=0,1,2,3,5,..,d} \tilde{\tilde{a}}^a \sqrt{\eta^{aa}}$, with α equal either to $d/2$ or to $(d-1)/2$ for an even and odd dimension d of the space, respectively. It can be checked that $(\tilde{\Upsilon})^2 = 1 = (\tilde{\tilde{\Upsilon}})^2$.

The projectors $P_\pm$ project out of any monomials of Eq.(4.1) the Grassmann odd and the Grassmann even part of the monomial, respectively. We find that for odd d the operators $\tilde{\Upsilon}$ and $\tilde{\tilde{\Upsilon}}$ coincide (up to $\pm i$ or ± 1) with $\tilde{\Gamma}$ and $\tilde{\tilde{\Gamma}}$ of Eq.(2.8), respectively.

We define two kinds of operators [2]. The first ones are binomials of operators forming the Grassmann odd Heisenberg algebra

$$S^{ab} := (\theta^a p^{\theta b} - \theta^b p^{\theta a}). \tag{4.6}$$

The second ones are binomials of operators forming the Clifford algebra

$$\tilde{S}^{ab} := -\frac{i}{4}[\tilde{a}^a, \tilde{a}^b], \;\; \tilde{\tilde{S}}^{ab} := -\frac{i}{4}[\tilde{\tilde{a}}^a, \tilde{\tilde{a}}^b], \tag{4.7}$$

with $[A, B] := AB - BA$.

Either S^{ab} or $\tilde{S}^{ab}$ or $\tilde{\tilde{S}}^{ab}$ fulfil the Lie algebra of the Lorentz group $SO(1, d-1)$ in the d-dimensional Grassmann space: $\{M^{ab}, M^{cd}\} = -i(M^{ad}\eta^{bc} + M^{bc}\eta^{ab} - M^{ac}\eta^{bd} - M^{bd}\eta^{ac})$, with M^{ab} equal either to S^{ab} or to $\tilde{S}^{ab}$ or to $\tilde{\tilde{S}}^{ab}$ and $M^{ab} = -M^{ba}$.

We see that

$$S^{ab} = \tilde{S}^{ab} + \tilde{\tilde{S}}^{ab}, \;\; \{\tilde{S}^{ab}, \tilde{\tilde{S}}^{cd}\} = 0 = \{\tilde{S}^{ab}, \tilde{\tilde{a}}^c\} = \{\tilde{a}^a, \tilde{\tilde{S}}^{bc}\}. \tag{4.8}$$

By solving the eigenvalue problem (see below) we find that operators $\tilde{S}^{ab}$, as well as the operators $\tilde{\tilde{S}}^{ab}$, define the fundamental or the spinorial representations of the Lorentz group, while $S^{ab} = \tilde{S}^{ab} + \tilde{\tilde{S}}^{ab}$ define the vectorial representations of the Lorentz group $SO(1, d-1)$.

Group elements are in any of the three cases defined by: $\mathcal{U}(\omega) = e^{\frac{i}{2}\omega_{ab}M^{ab}}$, where ω_{ab} are the parameters of the group.

Linear transformations, defined above, can then be written in terms of group elements as follows $\acute{\theta}^a = L^a{}_b\theta^b = e^{-\frac{i}{2}\omega_{cd}S^{cd}}\theta^a e^{\frac{i}{2}\omega_{cd}S^{cd}}$.

It can be proved for any d that M^2 is the invariant of the Lorentz group $\{M^2, M^{cd}\} = 0, \;\; M^2 = \frac{1}{2}M^{ab}M_{ab}$, and that for d=2n we can find the additional invariant Γ

$$\Gamma = \frac{i(-2i)^n}{(2n)!}\epsilon_{a_1a_2...a_{2n}}M^{a_1a_2}....M^{a_{2n-1}a_{2n}}, \;\; \{\Gamma, M^{cd}\} = 0, \tag{4.9}$$

where $\epsilon_{a_1a_2...a_{2n}}$ is the totally antisymmetric tensor with $2n$ indices and with $\epsilon_{123...2n} = 1$. This means that M^2 and Γ are for $d = 2n$ the two invariants or Casimir operators of the group $SO(d)$ (or $SO(1, d-1)$.) For $d = 2n+1$ the second invariant cannot be defined. (It can be checked that $\tilde{\Upsilon}$ and $\tilde{\tilde{\Upsilon}}$ of Eqs.(4.12) are the two invariants for the spinorial case for any d. For even d they coincide with $\tilde{\Gamma}$ and $\tilde{\tilde{\Gamma}}$, respectively, while for odd d the eigenvectors of these two operators are superpositions of Grassmann odd and Grassmann even monomials.)

While the invariant M^2 is trivial in the case when M^{ab} has spinorial character, since $(\tilde{S}^{ab})^2 = \frac{1}{4}\eta^{aa}\eta^{bb} = (\tilde{\tilde{S}}^{ab})^2$ and therefore M^2 is equal in both cases to the number $\frac{1}{2}\tilde{S}^{ab}\tilde{S}_{ab} = \frac{1}{2}\tilde{\tilde{S}}^{ab}\tilde{\tilde{S}}_{ab} = d(d-1)\frac{1}{8}$, it is a nontrivial differential operator in Grassmann space if M^{ab} have vectorial character ($M^{ab} = S^{ab}$). The invariant of Eq.(4.9) is always a nontrivial operator.

We assume that differentials of Grassmann coordinates $d\theta^a$ fulfil the Grassmann anticommuting relations [2, 3] $\{d\theta^a, d\theta^b\} = 0$ and we introduce a single integral over the whole interval of $d\theta^a$ $\int d\theta^a = 0, \;\; \int d\theta^a\theta^a =$

$1, a = 0,1,2,3,5,..,d$, and the multiple integral over d coordinates $\int d^d\theta^0\theta^1\theta^2\theta^3\theta^4...\theta^d = 1$, with $d^d\theta := d\theta^d...d\theta^3 d\theta^2 d\theta^1 d\theta^0$ in the standard way.

We define [2, 3] the inner product of two vectors $<\varphi|\theta>$ and $<\theta|\chi>$, with $<\varphi|\theta> = <\theta|\varphi>^*$ as follows:

$$<\varphi|\chi> = \int d^d\theta (\omega <\varphi|\theta>) <\theta|\chi>, \tag{4.10}$$

with the weight function $\omega = \prod_{k=0,1,2,3,..,d}(\frac{\partial}{\partial\theta^k} + \theta^k)$, which operates on the first function $<\varphi|\theta>$ only, and we define $(\alpha^{a_1 a_2 \cdots a_k}\theta^{a_1}\theta^{a_2}...\theta^{a_k})^+ = (\theta^{a_k}).....(\theta^{a_2})(\theta^{a_1})(\alpha^{a_1 a_2 \cdots a_k})^*$.

According to the above definition of the inner product it follows that $\tilde{a}^{a+} = -\eta^{aa}\tilde{a}^a$ and $\tilde{\tilde{a}}^{a+} = -\eta^{aa}\tilde{\tilde{a}}^a$, $(\tilde{a}^a\tilde{a}^b)^+ = -\eta^{aa}\eta^{bb}\tilde{a}^a\tilde{a}^b$, and $(\tilde{\tilde{a}}^a\tilde{\tilde{a}}^b)^+ = -\eta^{aa}\eta^{bb}\tilde{\tilde{a}}^a\tilde{\tilde{a}}^b$. The generators of the Lorentz transformations (Eqs.(4.7)) are self adjoint (if $a \neq 0$ and $b \neq 0$) or antiself adjoint (if $a = 0$ or $b = 0$) operators.

Either the volume element $d^d\theta$ or the weight function ω are invariants with respect to the Lorentz transformations (both are scalar densities of weight - 1).

According to Eqs.(4.2) and (4.6) (4.7)we find

$$S^{ab} = -i(\theta^a \frac{\partial}{\partial\theta_b} - \theta^b \frac{\partial}{\partial\theta_a}), \;\; \tilde{a}^a = (\frac{\partial}{\partial\theta_a} + \theta^a), \;\; \tilde{\tilde{a}}^a = i(\frac{\partial}{\partial\theta_a} - \theta^a),$$

$$\tilde{S}^{ab} = \frac{-i}{2}(\frac{\partial}{\partial\theta_a} + \theta^a)(\frac{\partial}{\partial\theta_b} + \theta^b), \;\; \tilde{\tilde{S}}^{ab} = \frac{i}{2}(\frac{\partial}{\partial\theta_a} - \theta^a)(\frac{\partial}{\partial\theta_b} - \theta^b), \; if a \neq b. \tag{4.11}$$

To find eigenvectors of any operator A, we solve the eigenvalue problem

$$<\theta|\tilde{A}_i|\tilde{\varphi}> = \tilde{\alpha}_i <\theta|\tilde{\varphi}>, \;\; <\theta|A_i|\varphi> = \alpha_i <\theta|\varphi>, \;\; i = \{1, r\}, \tag{4.12}$$

where $\tilde{A}_i$ and A_i stand for r commuting operators of spinorial and vectorial character, respectively.

To solve equations (4.12) we express the operators in the coordinate representation and write the eigenvectors as polynomials of θ^a. We orthonormalize the vectors according to the inner product, defined in Eq.(4.10), $<{}^a\tilde{\varphi}_i|{}^b\tilde{\varphi}_j> = \delta^{ab}\delta_{ij}, \quad <{}^a\varphi_i|{}^b\varphi_j> = \delta^{ab}\delta_{ij}$, where index a distinguishes between vectors of different irreducible representations and index j between vectors of the same irreducible representation. This determines the orthonormalization condition for spinorial and vectorial representations, respectively.

3 Lorentz Groups and Subgroups

The algebra of the group $SO(1, d-1)$ *or* $SO(d)$ contains [1] n subalgebras defined by operators $\tau^{Ai}, A = 1, n; i = 1, n_A$, where n_A is the number of elements of each subalgebra, with the properties

$$[\tau^{Ai}, \tau^{Bj}] = i\delta^{AB} f^{Aijk} \tau^{Ak}, \tag{4.13}$$

if operators τ^{Ai} can be expressed as linear superpositions of operators M^{ab}

$$\tau^{Ai} = c^{Ai}{}_{ab} M^{ab}, \;\; c^{Ai}{}_{ab} = -c^{Ai}{}_{ba}, \;\; A = 1, n, \;\; i = 1, n_A, \;\; a, b = 1, d. \tag{4.14}$$

Here f^{Aijk} are structure constants of the (A) subgroup with n_A operators. According to the three kinds of operators M^{ab}, two of spinorial and one of vectorial character, there are three kinds of operators τ^{Ai} defining subalgebras of spinorial and vectorial character, respectively, those of spinorial types being expressed with either $\tilde{S}^{ab}$ or $\tilde{\tilde{S}}^{ab}$ and those of vectorial type being expressed by S^{ab}. All three kinds of operators are, according to Eq.(4.13), defined by the same coefficients $c^{Ai}{}_{ab}$ and the same structure constants f^{Aijk}. ¿From Eq.(4.13) the following relations among constants $c^{Ai}{}_{ab}$ follow:

$$-4c^{Ai}{}_{ab}c^{Bjb}{}_{c} - \delta^{AB} f^{Aijk} c^{Ak}{}_{ac} = 0. \tag{4.15}$$

In the case when the algebra and the chosen subalgebras are isomorphic, that is if the number of generators of subalgebras is equal to $\frac{d(d-1)}{2}$, the inverse matrix e^{Aiab} to the matrix of coefficients $c^{Ai}{}_{ab}$ exists [1] $M^{ab} = \sum_{Ai} e^{Aiab}\tau^{Ai}$, with the properties $c^{Ai}{}_{ab}e^{Bjab} = \delta^{AB}\delta^{ij}, \;\; c^{Ai}{}_{cd}e^{Aiab} = \delta^a{}_c\delta^b{}_d - \delta^b{}_c\delta^a{}_d$.

When we look for coefficients $c^{Ai}{}_{ab}$ which express operators τ^{Ai}, forming a subalgebra $SU(n)$ of an algebra $SO(2n)$ in terms of M^{ab}, the procedure is rather simple [6, 2]. We find:

$$\tau^{Am} = -\frac{i}{2}(\tilde{\sigma}^{Am})_{jk}\{M^{(2j-1)(2k-1)} + M^{(2j)(2k)} + iM^{(2j)(2k-1)} - iM^{(2j-1)(2k)}\}. \tag{4.16}$$

Here $(\tilde{\sigma}^{Am})_{jk}$ are the traceless matrices which form the algebra of $SU(n)$. One can easily prove that operators τ^{Am} fulfill the algebra of the group $SU(n)$ for any of three choices for operators $M^{ab}: S^{ab}, \tilde{S}^{ab}, \tilde{\tilde{S}}^{ab}$.

In reference [2] coefficients $c^{Ai}{}_{ab}$ for a few cases interesting for particle physics can be found. Of special interest is the group $SO(1,13)$ with the subgroups $SO(1,3)$ and $SO(10) \supset SU(3)\times SU(2)\times U(1)$ which enables the unification of spins and charges. While the coefficients are the same for all three kinds of operators, the representations depend on the operators M^{ab}. After solving the eigenvalue problem (Eqs.(2.11)) for the invariants of the subgroups, the representations can be presented as polynomials of coordinates $\theta^a, a = 0,1,2,3,5,..,14$. The operators of spinorial character define the fermionic representations of the group and the subgroups, while the operators of vectorial character define the bosonic representations of the groups and the subgroups.

4 Lagrange Function for Free Particles in Ordinary and Grassmann Space and Canonical Quantization

We present in this section the Lagrange function for a particle which lives in a d-dimensional ordinary space of commuting coordinates and in a d-dimensional Grassmann space of anticommuting coordinates $X^a \equiv \{x^a, \theta^a\}$ and has its geodesics parametrized by an ordinary Grassmann even parameter (τ) and a Grassmann odd parameter(ξ). We derive the Hamilton function and the corresponding Poisson brackets and perform the canonical quantization, which leads to the Dirac equation with operators, which are differential operators in ordinary and in Grassmann space [1, 2].

$X^a = X^a(x^a, \theta^a, \tau, \xi)$ are called supercoordinates. We define the dynamics of a particle by choosing the action [1, 4] $I = \frac{1}{2}\int d\tau d\xi E E^i_A \partial_i X^a E^j_B \partial_j X^b \eta_{ab}\eta^{AB}$, where $\partial_i := (\partial_\tau, \overrightarrow{\partial}_\xi), \tau^i = (\tau, \xi)$, while E^i_A determines a metric on a two dimensional superspace τ^i , $E = det(E^i_A)$. We choose $\eta_{AA} = 0, \eta_{12} = 1 = \eta_{21}$, while η_{ab} is the Minkowski metric with the diagonal elements $(1,-1,-1,-1,...,-1)$. The action is invariant under the Lorentz transformations of supercoordinates: $X'^a = L^A{}_b X^b$. (See Eq.(4.3)). Since a supermatrix $E^i{}_A$ transforms as a vector in a two-dimensional superspace τ^i under general coordinate transformations of τ^i, $E^i{}_A \tau_i$ is invariant under such transformations and so is $d^2\tau E$. The action is locally supersymmetric.

Taking into account that either x^a or θ^a depend on an ordinary time parameter τ and that $\xi^2 = 0$, the geodesics can be described as a polynomial of ξ as follows: $X^a = x^a + \varepsilon\xi\theta^a$. We choose ε^2 to be equal either to $+i$ or to $-i$ so that it defines two possible combinations of supercoordinates. Accordingly we also choose the metric $E^i{}_A$: $E^1{}_1 = 1, E^1{}_2 = -\varepsilon M, E^2{}_1 = \xi, E^2{}_2 = N - \varepsilon\xi M$, with N and M Grassmann even and odd parameters, respectively. We write $\dot{A} = \frac{d}{d\tau}A$, for any A.

If we integrate the above action over the Grassmann odd coordinate $d\xi$, the action for a superparticle follows:

$$\int d\tau(\frac{1}{N}\dot{x}^a\dot{x}_a + \varepsilon^2\dot{\theta}^a\theta_a - \frac{2\varepsilon^2 M}{N}\dot{x}^a\theta_a). \tag{4.17}$$

Defining the two momenta

$$p^\theta_a := \frac{\overrightarrow{\partial} L}{\partial\dot{\theta}^a} = \epsilon^2\theta^a, \;\; p_a := \frac{\partial L}{\partial\dot{x}^a} = \frac{2}{N}(\dot{x}_a - Mp^{\theta a}), \tag{4.18}$$

the two Euler-Lagrange equations follow:

$$\frac{dp^a}{d\tau} = 0, \quad \frac{dp^{\theta a}}{d\tau} = \varepsilon^2\frac{M}{2}p^a. \tag{4.19}$$

Variation of the action(4.1) with respect to M and N gives the two constraints

$$\chi^1 := p^a a^\theta_a = 0, \chi^2 = p^a p_a = 0, \;\; a^\theta_a := ip^\theta_a + \varepsilon^2\theta_a, \tag{4.20}$$

while $\chi^3{}_a := -p^\theta_a + \epsilon^2\theta_a = 0$ (Eq.(4.18)) is the third type of constraints of the action(4.1). For $\varepsilon^2 = -i$ we find (Eq.(2.2)), that $a^\theta{}_a = \tilde{a}^a, \;\; \chi^3{}_a = \tilde{\tilde{a}}_a = 0$.

We find the generators of the Lorentz transformations for the action(4.17) to be (See also Eq.(4.6) and (4.7))

$$M^{ab} = L^{ab} + S^{ab} \; , \; L^{ab} = x^a p^b - x^b p^a \; , \; S^{ab} = \theta^a p^{\theta b} - \theta^b p^{\theta a} = \tilde{S}^{ab} + \tilde{\tilde{S}}^{ab}, \tag{4.21}$$

which show that parameters of the Lorentz transformations are the same in both spaces.

We define the Hamilton function:

$$H := \dot{x}^a p_a + \dot{\theta}^a p^\theta{}_a - L = \frac{1}{4}Np^a p_a + \frac{1}{2}Mp^a(\tilde{a}_a + i\tilde{\tilde{a}}_a) \tag{4.22}$$

and the corresponding Poisson brackets

$$\{A, B\}_p = \frac{\partial A}{\partial x^a}\frac{\partial B}{\partial p_a} - \frac{\partial A}{\partial p_a}\frac{\partial B}{\partial x^a} + \frac{\overrightarrow{\partial A}}{\partial\theta^a}\frac{\overrightarrow{\partial B}}{\partial p^\theta_a} + \frac{\overrightarrow{\partial A}}{\partial p^\theta_a}\frac{\overrightarrow{\partial B}}{\partial\theta^a}, \tag{4.23}$$

which have the properties of the generalized commutators [2].

If we take into account the constraint ${\chi^3}_a = \tilde{\tilde{a}}_a = 0$ in the Hamilton function (which just means that instead of H the Hamilton function $H + \sum_i \alpha^i \chi^i + \sum_a {\alpha^3}_a \chi^{3a}$ is taken, with parameters $\alpha^i, i = 1,2$ and ${\alpha^3}_a = -\frac{M}{2} p_a, a = 0,1,2,3,5,..,d$ chosen on such a way that the Poisson brackets of the three types of constraints with the new Hamilton function are equal to zero) and in all dynamical quantities, we find:

$$H = \frac{1}{4} N p^a p_a + \frac{1}{2} M p^a \tilde{a}_a, \;\; \chi^1 = p^a p_a = 0, \;\; \chi^2 = p^a \tilde{a}_a = 0, \tag{4.24}$$

$$\dot{p}_a = \{p_a, H\}_P = 0, \;\; \dot{\tilde{a}}_a = \{\tilde{a}_a, H\}_P = iMp_a, \tag{4.25}$$

which agrees with the Euler- Lagrange equations (4.19).

We further find

$$\dot{\chi}^i = \{H, \chi^i\}_P = 0, \;\; i = 1,2, \quad \dot{\chi^3}_a = \{H, {\chi^3}_a\}_P = 0, \;\; a = 0,1,2,3,5,..,d, \tag{4.26}$$

which guarantees that the three constraints will not change with the time parameter τ and that $\dot{\tilde{M}}^{ab} = 0$, with $\tilde{M}^{ab} = L^{ab} + \tilde{S}^{ab}$, saying that $\tilde{M}^{ab}$ is the constant of motion.

The Dirac brackets, which can be obtained from the Poisson brackets of Eq.(4.23) by adding to these brackets on the right hand side a term $-\{A, \tilde{\tilde{a}}^c\}_P \cdot (-\frac{1}{2i}\eta_{ce}) \cdot \{\tilde{\tilde{a}}^e, B\}_P$, give for the dynamical quantities, which are observables, the same results as the Poisson brackets. This is true also for $\tilde{a}^a$, ($\{\tilde{a}^a, \tilde{a}^b\}_D = i\eta^{ab} = \{\tilde{a}^a, \tilde{a}^b\}_P$), which is the dynamical quantity but not an observable since its odd Grassmann character causes supersymmetric transformations. We also find that $\{\tilde{a}^a, \tilde{\tilde{a}}^b\}_D = 0 = \{\tilde{a}^a, \tilde{\tilde{a}}^b\}_P$. The Dirac brackets give different results only for the quantities θ^a and $p^{\theta a}$ and for $\tilde{\tilde{a}}^a$ among themselves: $\{\theta^a, p^{\theta b}\}_P = \eta^{ab}, \{\theta^a, p^{\theta b}\}_D = \frac{1}{2}\eta^{ab}$, $\{\tilde{\tilde{a}}^a, \tilde{\tilde{a}}^b\}_P = 2i\eta^{ab}, \{\tilde{\tilde{a}}^a, \tilde{\tilde{a}}^b\}_D = 0$. According to the above properties of the Poisson brackets, I suggest that in the quantization procedure the Poisson brackets (4.23) rather than the Dirac brackets are used, so that variables $\tilde{\tilde{a}}^a$, which are removed from all dynamical quantities, stay as operators. Then $\tilde{a}^a$ and $\tilde{\tilde{a}}^a$ are expressible with θ^a and $p^{\theta a}$ (Eq.(4.2) and the algebra of linear operators introduced in Sect.2 can be used. We shall show, that suggested quantization procedure leads to the Dirac equation, which is the differential equation in ordinary and Grassmann space and has all desired properties.

In the proposed quantization procedure$-i\{A, B\}_p$ goes to either a commutator or to an anticommutator, according to the Poisson brackets (4.23). The operators $\theta^a, p^{\theta a}$ (in the coordinate representation they become $\theta^a \longrightarrow \theta^a, \; p^\theta_a \longrightarrow i\frac{\overrightarrow{\partial}}{\partial \theta^a}$) fulfil the Grassmann odd Heisenberg algebra, while the operators $\tilde{a}^a$ and $\tilde{\tilde{a}}^a$ fulfill the Clifford algebra.

The constraints (Eqs.(4.20)) lead to the Dirac like and the Klein-Gordon equations

$$p^a \tilde{a}_a |\tilde{\Psi}>= 0 \; , \; p^a p_a |\tilde{\Psi}>= 0, \text{ with } p^a \tilde{a}_a p^b \tilde{a}_b = p^a p_a. \tag{4.27}$$

Trying to solve the eigenvalue problem $\tilde{\tilde{a}}^a |\tilde{\Psi}>= 0, \;\; a = (0,1,2,3,5,...,d)$, we find that no solution of this eigenvalue problem exists, which means that the third constraint $\tilde{\tilde{a}}^a = 0$ can't be fulfilled in the operator form (although we take it into account in the operators for all dynamical variables in order that operator equations would agree with classical equations). We can only take it into account in the expectation value form

$$< \tilde{\Psi}|\tilde{\tilde{a}}^a|\tilde{\Psi} >= 0. \tag{4.28}$$

Since $\tilde{\tilde{a}}^a$ are Grassmann odd operators, they change monomials (Eq.(4.1)) of an Grassmann odd character into monomials of an Grassmann even character and opposite, which is the supersymmetry transformation. It means that Eq.(4.28) is fulfilled for monomials of either odd or even Grassmann character and that superpositions of the Grassmann odd and the Grassmann even monomials are not solutions for this system.

We can use the projector $P_\pm$ of Eq.(4.5) to project out of monomials either the Grassmann odd or the Grassmann even part. Since this projector commutes with the Hamilton function ($H = \frac{N}{4} p^a p_a + \frac{1}{2} M \; p^a \tilde{a}_a, \;\; \{P_\pm, H\} = 0$), it means that eigenfunctions of H, which fulfil the eq.(4.28), have either an odd or an even Grassmann character. In order that in the second quantization procedure fields $|\tilde{\Psi}>$ would describe fermions, it is meaningful to accept in the fermion case Grassmann odd monomials only.

We further see that although the operators $\tilde{a}^a$ fulfill Clifford algebra, they cannot be recognized as the Dirac $\tilde{\gamma}^a$ operator, since having an odd Grassmann character they transform fermions into bosons, which is not the case with the Dirac γ^a matrices. We therefore recognize the generators of the Lorentz transformations $-2i\tilde{S}^{bm}, \; m = 0,1,2,3$, with $b = 5$ as the Dirac γ^m operators. $\tilde{\gamma}^m = -\tilde{a}^5 \tilde{a}^m = -2i\tilde{S}^{5m}$, $m = 0,1,2,3$. (For another possible choice of the Dirac γ^m operators see the contribution to the discussions entitled "Can one connect the Dirac-Kaähler representation of Dirac spinors and spinor representations in Grassmann space, proposed by Mankoč?" written by Norma and Holger.)

We choose the Dirac operators $\tilde{\gamma}^a$ in the way which in the case that $<\tilde{\psi}|p^h|\tilde{\psi}>=0$, for $h \in \{5,d\}$, enables to recognize the equation

$$(\tilde{\gamma}^m p_m)|\tilde{\psi}>=0\ ,\ m=0,1,2,3. \tag{4.29}$$

as the Dirac equation for a massless particle. Since $-2i\tilde{S}^{5m}$ appear as $\tilde{\gamma}^m$, $SO(1,4)$ rather than $SO(1,3)$ is needed to describe the spin degrees of freedom of fermionic fields. It can be checked that $\tilde{\gamma}^m$ fulfill the Clifford algebra $\{\tilde{\gamma}^m,\tilde{\gamma}^n\}=\eta^{mn}$, while $\tilde{S}^{mn}=-\frac{i}{4}[\tilde{\gamma}^m,\tilde{\gamma}^n]_-, m \in \{0,3\}$. Accordingly also the group $SO(1,14)$ instead of $SO(1,13)$ is needed to unify spins and charges. (In the Norma and Holger contribution the $\tilde{\gamma}^a = i\tilde{a}^a\tilde{\tilde{a}}^0$ (Eq.30) are suggested as the Dirac γ^a operators, having all the needed properties. In this case the additional coordinate θ^5 is not needed.)

We presented in Ref. [2] four Dirac four spinors (the polynomials of θ^a) which fulfill Eq.(4.12).

For large enough d not only do generators of Lorentz transformations in Grassmann space define the spins of fields in the four dimensional subspace, they also define the electromagnetic, the weak and the colour charges. This is true, for example, for $d=13$, since $SO(1,13)$ has the subalgebra $SO(1,3)\times SO(10)$, while $SO(10)$ has the subalgebra $SU(3)\times SU(2)\times U(1)$. In this case $\tilde{\tau}^{Ai}$ are linear superpositions of operators $\tilde{S}^{ab},\ a,b \in \{5,d\}$ fulfilling the algebras as presented in Eqs.(3.1-3.3) and defining the algebras of $SU(3), SU(2), U(1)$, while $SO(1,3)$ remains to define the spin degrees of freedom in the four dimensional subspace. We find the spinorial representations of the corresponding Casimir operators as functions of θ^a determining weak charge doublets, colour charge triplets and electromagnetic charge singlets [2].

5 Particles in Gauge Fields

The dynamics of a point spinning particle in gauge fields, the gravitational and the Yang-Mills fields, can be obtained by transforming in the Lagrangean vectors from a freely falling to an external coordinate system [5]. To do this, supervielbeins $\mathbf{e}^{ia}{}_{\mu}$ have to be introduced, which in our case depend on ordinary and on Grassmann coordinates, as well as on two types of parameters $\tau^i=(\tau,\xi)$. Since there are two kinds of derivatives ∂_i, there are two kinds of vielbeins [1, 2]. The index a refers to a freely falling coordinate system (a Lorentz index), the index μ refers to an external coordinate system (an Einstein index). Vielbeins with a Lorentz index smaller than five will determine ordinary gravitational fields. Spin connections appear in the theory as (a part of) Grassmann odd fields. Those with a Lorentz index higher than four define Yang-Mills fields.

We write the transformation of vectors as follows $\partial_i X^a = \mathbf{e}^a{}_{\mu}\partial_i X^{\mu}$, $\partial_i X^{\mu} = \mathbf{f}^{\mu}{}_a \partial_i X^a$, $\partial_i = (\partial_\tau, \partial_\xi)$. ¿From here it follows that $\mathbf{e}^a{}_{\mu}\mathbf{f}^{\mu}{}_b = \delta^a{}_b$, $\mathbf{f}^{\mu}{}_a\mathbf{e}^a{}_{\nu} = \delta^{\mu}{}_{\nu}$.

Again we make a Taylor expansion of vielbeins with respect to ξ : $\mathbf{e}^a{}_{\mu} = e^a{}_{\mu} + \varepsilon\xi\theta^b e^a{}_{\mu b}$, $\mathbf{f}^{\mu}{}_a = f^{\mu}{}_a - \varepsilon\xi\theta^b f^{\mu}{}_{ab}$.

Both expansion coefficients again depend on ordinary and on Grassmann coordinates. Having an even Grassmann character, $e^a{}_{\mu}$ will describe the spin 2 part of a gravitational field. The coefficients $\varepsilon\theta^b e^a{}_{\mu b}$ have an odd Grassmann character ($\varepsilon=-i$, so that $\tilde{\tilde{a}}^a=0$). They define the spin connections [1, 2].

It follows that $e^a{}_{\mu}f^{\mu}{}_b = \delta^a{}_b$, $f^{\mu}{}_a e^a{}_{\nu} = \delta^{\mu}{}_{\nu}$, $e^a{}_{\mu b}f^{\mu}{}_c = e^a{}_{\mu}f^{\mu}{}_{cb}$.

We find the metric tensor $\mathbf{g}_{\mu\nu} = \mathbf{e}^a{}_{\mu}\mathbf{e}_{a\nu}$, $\mathbf{g}^{\mu\nu} = \mathbf{f}^{\mu}{}_a\mathbf{f}^{\nu a}$.

We use the notation $e^a{}_{\nu,\mu^x} = \frac{\partial}{\partial x^{\mu}}e^a{}_{\nu}$, $\overrightarrow{e^a}_{\nu,\mu^\theta} = \frac{\overrightarrow{\partial}}{\partial\theta^{\mu}}e^a{}_{\nu}$. Rewriting the action from Sect.4 in terms of an external coordinate system, using the Taylor expansion of supercoordinates X^{μ} and superfields $\mathbf{e}^a{}_{\mu}$ and integrating the action over the Grassmann odd parameter ξ, the action

$$I = \int d\tau\{\frac{1}{N}g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}\ -\ \epsilon^2\frac{2M}{N}\theta_a e^a{}_{\mu}\dot{x}^{\mu}\ +\ \varepsilon^2\frac{1}{2}(\dot{\theta}^{\mu}\theta_a - \theta_a\dot{\theta}^{\mu})e^a{}_{\mu} +$$

$$+\ \varepsilon^2\frac{1}{2}(\theta^b\theta_a - \theta_a\theta^b)e^a{}_{\mu b}\dot{x}^{\mu}\}, \tag{4.30}$$

defines the two momenta of the system $p_{\mu} = \frac{\partial L}{\partial\dot{x}^{\mu}} = p_{0\mu} + \frac{1}{2}\tilde{S}^{ab}e_{a\mu b}$, $p^{\theta}_{\mu} = -i\theta_a e^a{}_{\mu} = -i(\theta_{\mu} + \overrightarrow{e}^a{}_{\nu,\mu_\theta}e_{a\alpha}\theta^{\nu}\theta^{\alpha})$, ($\varepsilon^2=-i$). Here $p_{0\mu}$ are the covariant (canonical) momenta of a particle. For $p^{\theta}_a = p^{\theta}_{\mu}f^{\mu}{}_a$ it follows that p^{θ}_a is proportional to θ_a. Then $\tilde{a}_a = i(p^{\theta}_a - i\theta_a)$, while $\tilde{\tilde{a}}_a=0$. We may further write

$$p_{0\mu} = p_{\mu} - \frac{1}{2}\tilde{S}^{ab}e_{a\mu b} = p_{\mu} - \frac{1}{2}\tilde{S}^{ab}\omega_{ab\mu}\ ,\ \omega_{ab\mu} = \frac{1}{2}(e_{a\mu b} - e_{b\mu a}), \tag{4.31}$$

which is the usual expression for the covariant momenta in gauge gravitational fields [5]. One can find the two constraints

$$p^{\mu}_0 p_{0\mu} = 0 = p_{0\mu}f^{\mu}{}_a\tilde{a}^a. \tag{4.32}$$

To see how Yang-Mills fields enter into the theory, the Dirac-like equation (4.32) has to be rewritten in terms of components of fields which determine gravitation in the four dimensional subspace and of those which

determine gravitation in higher dimensions, assuming that the coordinates of ordinary space with indices higher than four stay compacted to unmeasurable small dimensions. Since Grassmann space manifests itself through average values of observables only, compactification of a part of Grassmann space has no meaning. However, since parameters of Lorentz transformations in a freely falling coordinate system for both spaces have to be the same, no transformations to the fifth or higher coordinates may occur at measurable energies. Therefore, the four dimensional subspace of Grassmann space with the generators defining the Lorentz group $SO(1,3)$ is (almost) decomposed from the rest of the Grassmann space with the generators forming the (compact) group $SO(d-4)$, because of the decomposition of ordinary space. This is valid on the classical level only.

We shall assume the case in which only some components of fields differ from zero:

$$\left(\begin{array}{c|c} e^m{}_\alpha & 0 \\ \hline 0 & e^h{}_\sigma \end{array}\right), \quad \alpha, m \in (0,3),\ \sigma, h \in (5,d),\ i \in (1,2), \tag{4.33}$$

while vielbeins $e^m{}_\alpha, e^k{}_\sigma$ depend on θ^a and $x^\alpha, \alpha \in \{0,3\}$, only. Accordingly we have only $\omega_{ab\alpha} \neq 0$. We recognize, as in the freely falling coordinate system, that Grassmann coordinates with indices from 0 to 3 determine spins of fields, while Grassmann coordinates with indices higher or equal to 5 determine charges of fields. We shall take $< p^h >= 0$, $a \geq 5$. We find

$$\tilde{\gamma}^a f^\mu{}_a p_{0\mu} = \tilde{\gamma}^m f^\alpha{}_m (p_\alpha - \frac{1}{2}\tilde{S}^{mn}\omega_{mn\alpha} + A_\alpha), \text{ with } A_\alpha = \sum_{A,i} \tilde{\tau}^{Ai} A^{Ai}_\alpha \tag{4.34}$$

and $\sum_{A,i} \tilde{\tau}^{Ai} A^{Ai}_\alpha = \frac{1}{2}\tilde{S}^{hk}\omega_{hk\alpha}$, $h,k = 5,6,7,8,..d$.

As we already stated in Sect.3 for $d = 14$, $SO(1,13)$ has the subalgebras $SO(1,3) \times SO(10)$, while $SO(10)$ has the subalgebras $SU(3) \times SU(2) \times U(1)$.

Therefore, in Eq.(4.34) the fields $\omega_{hk\alpha}$ determine all the Yang-Mills fields, including electromagnetic ones. The proposed unification differs from the Kaluza-Klein types of unification, since Yang-Mills fields are not determined by nondiagonal terms of vielbeins $e^h{}_\alpha$. Instead they are determined by spin connections and it seems that in the proposed theory there is no difficulties with the Planck mass of the electron unless spin connections or vielbeins are supposed to generate the Yukawa nonzero masses of fermions.

Torsion and curvature follow from the Poisson brackets $\{p_{0a}, p_{0b}\}_p$, with $p_{0a} = f^\mu{}_a(p_\mu - \frac{1}{2}\tilde{S}^{cd}\omega_{cd\mu})$. We find $\{p_{0a}, p_{0b}\}_p = -\frac{1}{2}S^{cd}R_{cdab} + p_{0c}T^c{}_{ab}$, $R_{cdab} = f^\mu{}_{[a}f^\nu{}_{b]}(\omega_{cd\nu,\mu^x} + \omega_c{}^e{}_\mu\omega_{ed\nu} + \overrightarrow{\omega}_{cd\mu,f^\theta}\theta^e\omega_e{}^f{}_\nu)$, $T^c{}_{ab} = e^c{}_\mu(f^\nu{}_{[b}f^\mu{}_{a],\nu} + \omega_{e\nu}{}^d\theta^e f^\nu{}_{[b}\overrightarrow{f^\mu}{}_{a],d^\theta})$, with $A_{[a}B_{b]} = A_aB_b - A_bB_a$. For $e^m{}_\alpha = \delta^m{}_\alpha$ one easily sees that Eq.(5.9) manifests the Dirac equation for a particle with Yang-Mills charges in external fields.

6 Concluding Remarks

In this talk the theory in which space has d ordinary and d Grassmann coordinates was presented. Two kinds of generators of Lorentz tranformations in Grassmann space can be defined. The generators of spinorial character define the spinorial representations of the Lorentz group, the generators of the vectorial character define the vectorial representations of the Lorentz group. Both kinds of generators are the linear differential operators in Grassmann space. The Lorentz group $SO(1,d-1)$ contains for $d = 13$ as subgroups $SO(1,3), SU(3), SU(2)$ and $U(1)$. While $SO(1,3)$ defines spins of fermionic and bosonic fields, define $SU(3), SU(2)$ and $U(1)$ charges of both fields. Charges of fermionic fields belong to the spinorial representations, while charges of bosonic fields belong to the vectorial representations.

When looking for the representations of the operators $\tilde{S}^{mn}$, $m,n \in 0,3$ as polynomials of θ^a, $a \in 0,3$ and operators $\tilde{\tau}^{Ai}$ as polynomials of θ^h, $a \in 5,13$, we find representations of the group $SO(1,13)$ as outer products of the representations of subgroups. The Grassmann odd polynomials, which are the Dirac four spinors, are triplets or singlets with respect to the colour charge, doublets or singlets with respect to the weak charge and may have hypercharge [2] equal to $\pm\frac{1}{6}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{2}, \pm 1, 0$. When looking for the representations of $SO(1,13)$, as Grassmann even polynomials of θ^a, in terms of the subgroups $SO(1,3), SU(3), SU(2), U(1)$, we find scalars and vectors, which are singlets and octets with respect to the colour charge, triplets and singlets with respect to the weak charge and may have the hypercharge equal to 0 or to ± 1. We also find scalars, which are weak charge doublets. These representations are presented in Ref. [2]. I shall discuss properties of representations in discussions sections. I shall show that the theory may offer the answer to some of open problems of the Standard electroweak model: The approach suggests four rather than three families of quarks and leptons, it predicts the left handed weak charge doublets together with right handed weak charge singlets in the same fermionic multiplet.

We presented the Lagrange function for a particle living on a supergeodesics, with the momentum in the Grassmann space proportional to the Grassmann coordinate. In the quantization procedure the Dirac equation follows, with γ^a operators, which have the even Grassmann character and are differential operators in Grassmann space with coordinates $\theta^a, \ a \in 0, 3$. When transforming the Lagrange function from the freely falling to the external coordinate system, vielbeins and spin connections describe not only the gravitational field but also the Yang-Mills fields. Since the generators of the Lorentz transformations with indices higher than four determine charges of particles and spin connections again with indices higher than four describe the Yang-Mills fields (rather than vielbeins with one index smaller than four and another greater than three as in the Kaluza-Klein theories), the problem of the Kaluza-Klein theories, which is the Planck mass of charged particles, seems not to occur.

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Masses and Mixing Angles and Going beyond the Standard Model

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Abstract

The idea of following Michel and O'Raifeartaigh of assigning meaning to the (gauge) <u>group</u> and not only the Lie algebra for a Yang Mills theory is reviewed. Hints from the group and the fermion spectrum of the Standard Model is used to suggest the putting forward of our AGUT-model, which gives a very good fit of the orders of magnitudes of the quark and lepton masses and the mixing angles including the CP-breaking phase. But for neutrino oscillations modifications of the model is needed. Baryogenesis is not in conflict with the model. Masses and mixing angles and going beyond the Standard Model

1 Introduction

For the purpose of finding out what comes beyond the Standard Model it is unfortunate that the latter works so exceedingly well that it actualy describes satisfactorily almost all we know and can make experiments about: Just extending with even classical Einsteinian gravity is sufficient to provide well working laws of nature for all to day practical purposes. So the true hints for going beyond the Standard Model can except for pure theoretical estetical arguments only come from the structure and parameters - which are not yet understood inside the Standard Model - of the Standard Model or from the extremely little information we have about the physics outside the below 1 TeV range where so far Standard Model could potentially work perfectly. The extremely little knowledge we have for the very short distances comes from the baryon number being presumably not conserved: 1) If baryon number assymetry should be cosmologically produced at the weak scale and not for instance be due to a $B-L$ assymetry from earlier time we would need some new physics, and even if it was an earlier $B-L$ assymetry that caused the observed baryon number there would at some scale at least have to be produced the $B-L$ or it would have to truly primordial. 2) The lack of proton decay gives information that e.g. a naive SU(5) GUT is not correct. Finally we really do see nonstandard model physivcs in the neutrino oscillations.

But apart from these tiny bits of information we mainly have the structure and coupling constants and masses in the Standard Model from which to seek to guess the model beyond!

We (Svend Erik Rugh et al.) estimated that the amount of information in these parameters as measured so far and in the Standard model structure was just around a couple of hundred bits. It could all be written on one line.

What is now the inspiring information on this line?

In section 2 we stress how part of the information about the quantum numbers of the quarks and the leptons (really their Weyl components) can be packed into saying what <u>group</u> rather than only what Lie algebra is to be represented.

In section 3 we look at another hint : the large mass ratios of the quark and lepton masses in the various generatios, and the small mixing angles.

With good will these hints could be taken to point in the direction of the AGUT gauge group whhich is our favorite model. AGUT stands for anti-grand unification and is indeed in a way to be explained based on assumptions oppsite to the ones leading to the usual SU(5) GUT.

In section 4 we put forward the model, especially the gauge group AGUT which we characterize as largest group not unifying the irreducible fermion representations of the Standard Model. AGUT stands for Anti Grand Unification which is the name we give to the gauge group $SMG^3 \times U(1)_f$, where $SMG \equiv SU(3) \times SU(2) \times U(1)$.

The Higgs fields responsible for breaking the AGUT gauge group $SMG^3 \times U(1)_f$ to the diagonal SMG subgroup, identified as the SM gauge group, are considered in section 5.

The structure of the resulting fermion mass matrices are presented in section 6, together with details of a fit to the charged fermion spectrum. In sections 7 and 8, we briefly discuss the problems of baryogenesis and neutrino oscillations respectively. Finally we mention the relation to the MPP principle - see the contribution by Larisa Laperashvili - in section 9 and the conclusion is in section 10.

2 Gauge Group

Since the Standard Model is a Yang Mills theory the gauge lie algebra is an important structural element to specify in order to specify the model, and this structure is presumably of some significant informative strength as far as it seems not so obvious to say why the theory working at the present stage of the experiments should just have this gauge algebra: counting all the many cross products of various Lie algebras it is not so immediately clear why it should be the algebra corresponding to $U(1) \times SU(2) \times SU(3)$ that should be the God-selected.

Refering to the works by Michel and Oraifaightaigh [2] we have long suggested to consider rather than the gauge lie algebra - which is of course what specifies the couplings of the Yang Mills fields to each other - the gauge group. A priori the gauge group only is relevant as far as its Lie algebra determines the couplings of the Yang Mills particles or fields , the coupling constants being proportional to the structure constants of the Lie algebra. If there were a truly ontologically existing lattice it would be a different matter because in that case there would be place for specifying a group and not only the to the group corresponding Lie algebra. There is , however, also a phenemenologically accessible way of asigning a meaning to the gauge group and not only the algebra: Different gauge groups with the same Lie algebra allow a different set of matter field representations. Certain groups are thus not allowed if one requires that the experimentally found matter shall be represented under the group.

Now the connection between Lie algebra and Lie group is so that there are several groups corresponding to one algebra in general but always only one algebra to each group. Considering only connected groups as is reasonable here there corresponds to each Lie algebra a unique group, the covering group characterized by being simply connected - i.e. that any closed curve on it can be continuosly contracted to a point - from which which all the other connected groups with the given Lie algebra can be obtained by dividing out of the covering group the various dicrete invariant subgroups of it. Now it is mathematically so that all representations of the Lie algebra are also representations of the covering group, but for the other groups with the given Lie algebra it is only some of the algebra representations that are also representations of the group. You can therefore never exclude that the covering group can be used, whatever the matter field representations may be, while many of the other groups can easily be excluded whenever some matter field representation is known. If one has found a large number of matter fields as $\{(2\pi, -1, exp(2\pi/3)1)^n | n \in \mathbf{Z})\}$ is the case in the Standard Model then it might be almost remarkable if any group other than the covering group has all these representations. For the Standard Model it can in fact rather easily be computed that there is remarkably enough a group other than the covering one which contains all the representations found in nnature so far ! In the light of the relatively "many" matter representations we could then claim that there is a phenomenological evidence for that this group is the GROUP of select by nature or the Standard Model group, which we write by short hand SMG. Indeed the the group that in this way deserves to be called the Standard Model Group is $SMG = S(U(2) \times U(3)) = (\mathbf{R} \times SU(2) \times SU(3))/\{(2\pi, -1, exp(2\pi/3)1)^n | n \in \mathbf{Z})\}$. It may be described as the subset of the cross product of $U(2)$ and $U(3)$ for which the product of the determinat for the U(2) group element conceived of as a matrix and that of SU(3) is unity.

What this putting forward of a special group really means is that a regularity in the system of matter field representations that occur phenemenologically can be expressed by the group statement. In the case of the Standard Model Group $SMG = S(U(2) \times U(3))$ =$(\mathbf{R} \times SU(2) \times SU(3))/\{(2\pi, -1, exp(2\pi/3)1)^n | n \in \mathbf{Z})$ it is actually the regularity required by the wellknown rules for electric charge quantization that can be expressed as the requirement of the repesentations in nature being representations not only of the Lie algebra but really of this group. The electric charge quantization rule is:

For the colorless particles we have the Milikan charge quantization of all charges being integer when measured in units of the elementary charge unit, but for colored particles the charge deviate from being integer by $-1/3$ elementary charge for quarks and by $+1/3$ for antiquarks.

This rule can be expressed by introducing the concept of triality t, which characterizes the representation of the center $\{exp(ni2\pi/3)1^{3\times3} | n = 0, 1, 2\} \subset SU(3)$ writting under the representation in question and is defined so that $t = 0$ for the trivial representation or for decuplets , octets and so on , while it is $t = 1$ for triplet ($\underline{3}$) or anti sixplets etc. and $t = -1$ for antitriplet ($\overline{\underline{3}}$) or sixtet etc. Then it is written

$$Q + t/3 = 0(\text{mod}1) \tag{5.1}$$

where Q is the electric charge $Q = y/2 + t_3/2$ (here t_3 is the third component of the weak isospin (SU(2)), and y is the weak hypercharge). We may write this charge quantization rule as

$$y/2 + d/2 + t/3 = 0(\text{mod}1) \tag{5.2}$$

where we have introduced the duality d which is defined to be 0 when the weak isospin is integer and $d = 1$ when it is half integer. It is namely then easily seen that $d/2 = t_3/2(\text{ mod}1)$ for all weak isospin representations.

Now the point is that this restriction on the representations ensure that the subgroup $\{(2\pi, -1, exp(2\pi/3)1)^n | n \in \mathbf{Z})$ is represented trivially and that thus the representations allowed really are representations of the group $SMG = S(U(2) \times U(3)) =$
$= (\mathbf{R} \times SU(2) \times SU(3))/\{(2\pi, -1, exp(2\pi/3)1)^n | n \in \mathbf{Z})$.

After having made sense of the group it would be natural to ask if this group could somehow give us a hint about what goes on beyond the Standard Model. Brene and one of us ([3],[4]) have argued for two indications comming out the group-choice of nature:

a) The charge quantization rule in the Standard Model is in some sense linking the invariant sub Lie algebras more strongly than in - in a certain way of counting - any other group would do. To be more specific : There are six different combinations of triality and duality - i.e. really of classes of representations of the non-abelian part of the gauge Lie algebra - that can be specified by providing the abelian charge $y/2$. The logarithm of this number of such classes divided by the dimension of the Cartan algebra four in the case of the Standard Model group is larger for the SMG than for any other group (except cross products of SMG with itself, for which the mentioned ratio must be the same value). We called this ratio χ.

b) The SMG has rather few automorphisms and can be considered to a large extend specified as one of the most "skew" groups.

If you would take the point a) to help to guess some group beyond the Standard Model you could say that we should expect also the group behind to have the ratio *chi* large for the group behind. That requirement points in the direction of having a cross product power of the standard model group, because such a cross product has just the same *chi* as the standard model group itself.a

3 The large mass ratios of leptons and quarks

What is the origin of the well-known pattern of large ratios between the quark and lepton masses and of the small quark mixing angles? This is the problem of the hierarchy of Yukawa couplings in the Standard Model (SM). We suggest [1] that the natural resolution to this problem is the existence of some approximately conserved chiral charges beyond the SM. These charges, which we assume to be gauged, provide selection rules forbidding the transitions between the various left-handed and right-handed fermion states (except for the top quark).

For example, we suppose that there exists some charge (or charges) Q for which the quantum number difference between left- and right-handed Weyl states is larger for the electron than for muon:

$$|Q_{eL} - Q_{eR}| > |Q_{\mu L} - Q_{\mu R}| \tag{5.3}$$

It then follows that the SM Yukawa coupling for the electron g_e is suppressed more than that for the muon g_μ, when Q is taken to be approximately conserved. This is what is required if we want to explain the electron-muon mass ratio.

We shall take the point of view that, in the fundamental theory beyond the SM, the Yukawa couplings allowed by gauge invariance are all of order unity and, similarly, all the mass terms allowed by gauge invariance are of order the fundamental mass scale of the theory—say the Planck scale. Then, apart from the matrix element responsible for the top quark mass, the quark-lepton mass matrix elements are only non-zero due to the presence of other Higgs fields having vacuum expectation values (VEVs) smaller (typically by one order of magnitude) than the fundamental scale. These Higgs fields will, of course, be responsible for breaking the fundamental gauge group G- what ever it may be - down to the SM group. In order to generate a particular effective SM Yukawa coupling matrix element, it is necessary to break the symmetry group G by a combination of Higgs fields with the appropriate quantum number combination $\Delta\vec{Q}$. When this "$\Delta\vec{Q}$" is different for two matrix elements they will typically deviate by a large factor. If we want to explain the observed spectrum of quarks and leptons in this way, it is clear that we need charges which—possibly in a complicated way—separate the generations and, at least for $t - b$ and $c - s$, also quarks in the same generation. Just using the usual simple $SU(5)$ GUT charges does not help, because both (μ_R and e_R) and (μ_L and e_L) have the same $SU(5)$ quantum numbers. So we prefer to keep each SM irreducible representation in a separate irreducible representation of G and introduce extra gauge quantum numbers distinguishing the generations, by adding extra cross-product factors to the SM gauge group.

What the structure of the quark and lepton spectrum really calls for is separation between generations and also between at least the c-quark and t-quark from their generations even. Unification is trictly speaking not called for because it is wellknown that the simplest $SU(5)$ unification can only be made to work by either having complicated Higgs fields replacing the simple Weinberg salam Higgs field taken as a fiveplet - Georgy-Jarlskog model - or even more sophisicated mechanisms involving other SU(5) breaking than in the minimal SU(5). The experimental mass ratios predicted by simple SU(5) may work for the case of τ and b-quark adjusted by susy or a reasonable scale, but then the μ to s and the e to d cases do not agree with such a simple SU(5) with only the fiveplet Higgsfield(or two if supersymmetric) replacing the Weinberg Salam Higgs.

In other words it is called for separation not unification!

Although one of us (Colin Froggatt) in his contribution shows the possibility of extending the standard model with just two extra $U(1)$ groups and get a fit of the quark and lepton mass order of magnitudes - actually that model is the model of the present contribution with the nonabelian groups amputated away - it is , if one insisists on quantum numbers closer to be minimal/small relative to what is allowed by the quantization rules (what is allowed by the requirement of representing the group), suggested to be better to have a larger group extending the SMG.

4 The "maximal" AGUT gauge group

To limit the search for the gauge group beyond the Standard Model let us take the point of view that we do not look for the whole gauge group G say , but only for that factorgroup $G' = G/H$ which transform the already known quark and lepton weyl fields in a nontrivial way. That is to say we ask for the group obtained by dividing out the subgroup $H \subset G$ which leaves the quark and lepton fields unchanged. This factor group G' can then be identified with its representation on the Standard model fermions, i.e. as a subgroup of the the $U(45)$ group of all possible unitary transformations of the 45 Weyl fields for the Standard model. If one took as G one of the extensions of SU(5) such as SO(10) or the E-groups which are promising unification groups, the factor group G/H would be SU(5) only, the extension parts can be said to only transform particles that are not in the standard model (and thus could be pure phantasy a priori). We would like to assume that there shall be no gauge or mixed anomalies. So now we can ask to add some further suggestive properties for G' that could help us choosing it:

If we ask the smallest extension of the Standard Model unifying as many as possible of the under the standard model irreducible representations to irreducible representations under G' we get as can be relatively easily seen $SU(5)$ the usual way. That represents all the SO(10) and E-groups, since we here talked about having divided out the part H that transform the known particles trivially.

But as we above argued there rather from the empirical indicators a call for the opposite: separation and a big group!

We have actually calculated that among the subgroups of the $U(45)$ of unitary transformations of the Standard model Weyl fermions without anomalies the biggest seperating group is the AGUT-group which is the gauge group of the model put forward here.

The AGUT model is based on extending the SM gauge group $SMG = S(U(2) \times U(3))$ in a similar way to grand unified $SU(5)$, but rather to the non-simple $SMG^3 \times U(1)_f$ group.

The $SMG^3 \times U(1)_f$ group should be understood that, near the Planck scale, there are three sets of SM-like gauge particles. Each set only couples to its own proto-generation [e.g. the proto- u, d, e and ν_e particles], but not to the other two proto-generations [e.g. the proto- c, s, μ, ν_μ, t, b, τ and ν_τ particles]. There is also an extra abelian $U(1)_f$ gauge boson, giving altogether $3 \times 8 = 24$ gluons, $3 \times 3 = 9$ W's and $3 \times 1 + 1 = 4$ abelian gauge bosons. The couplings of the $SMG_i = S(U(2) \times U(3))_i \approx SU(3)_i \times SU(2)_i \times U(1)_i$ group to the i'th proto-generation are identical to those of the SM group. Consequently we have a charge quantisation rule,for each of the three proto-generation weak hypercharge quantum numbers y_i.

To first approximation—namely in the approximation that the the quark mixing angles V_{us}, V_{cb}, V_{ub} are small—we may ignore the prefix *proto*$-$. However we really introduce in our model some "proto-fields" characterized by their couplings to the 37 gauge bosons of the $SMG^3 \times U(1)_f$ group. The physically observed u-quark, d-quark etc. are then superpositions of the proto-quarks (or proto-leptons), with the same named proto-particle dominating. Actually there is one deviation from this first approximation rule that proto-particles correspond to the same named physical particle. In the AGUT fit to the quark-lepton mass spectrum, discussed below, we find that to first approximation the right-handed components of the top and the charm quarks must be permuted:

$$c_{R\ PROTO} \approx t_{R\ PHYSICAL} \qquad t_{R\ PROTO} \approx c_{R\ PHYSICAL} \tag{5.4}$$

But for all the other components we have:

$$t_{L\ PROTO} \approx t_{L\ PHYSICAL} \qquad b_{R\ PROTO} \approx b_{R\ PHYSICAL} \tag{5.5}$$

and so on.

The AGUT group breaks down an order of magnitude or so below the Planck scale to the SM group as the diagonal subgroup of its SMG^3 subgroup.

For this breaking we shall fit a relatively complicated sytem of Higgses with names W, T, ξ, and S. In order to get neutrino masses fitted we need an even more complicated system. See the thesis of Mark Gibsosn.

It should however, be said that although at the very hing energies just under the Planck energy each generation has its own gluons own W's etc. then the breaking makes only one linear combination of a certain

color combination of gluon say "survive" down to the low energy and below the ca 1/10 of the Planck scale it is only these linear combinations that are present and thus the couplings of the gauge particles - namely at low energy only these combinations - are the same for all three generations.

You can also say that the phenomenological gluon say is a linear combination with amplitude $1/\sqrt{3}$ for each of the AGUT-gluons of the same color combination. That then also explains that the coupling constat for the phenomenological gluon coules with a strength that is $\sqrt{3}$ times smaller, if as we effectively assume the three AGUT $SU(3)$ coulings were equal to each other. Really the formula connecting the fine structure constant for the in our model to low energy surviving diagonal subgroup $\{(U,U,U)|U \in SMG\} \subseteq SMG^3$ is

$$\frac{1}{\alpha_{diag,i}} = \frac{1}{\alpha_{\text{1st gen.},i}} + \frac{1}{\alpha_{\text{2nd gen.},i}} + \frac{1}{\alpha_{\text{3rd gen.},i}} \tag{5.6}$$

. Here the index i is meant to run over the three groups ina SMG, namely $i = U(1), SU(2), SU(3)$, so that e.g. $i = 3$ means that we talk abou the gluon couplings (of the generation in question).

The gauge coupling constants do not, of course, unify, because we not built the groups U(1) , SU(2) and SU(3) together, but their values have been successfully calculated using the so-called multiple point principle [5], which is a further assumption we put into the model (see for this (also) Larisa Laperashvilis contribution to these proceedings).

At first sight, this $SMG^3 \times U(1)_f$ group with its 37 generators seems to be just one among many possible SM gauge group extensions.

However, we shall now argue it is not such an arbitrary choice, as it can be uniquely specified by postulating 4 reasonable requirements on the gauge group G beyond the SM. As a zeroth postulate, of course, we require that the gauge group extension must contain the Standard Model group as a subgroup $G \supseteq SMG$. In addition it should obey the following 4 postulates:

The first two are also valid for $SU(5)$ GUT:

1. G should transform the presently known (left-handed, say) Weyl particles into each other, so that $G \subseteq U(45)$. Here $U(45)$ is the group of all unitary transformations of the 45 species of Weyl fields (3 generations with 15 in each) in the SM.

2. No anomalies, neither gauge nor mixed. We assume that only straightforward anomaly cancellation takes place and, as in the SM itself, do not allow for a Green-Schwarz type anomaly cancellation [6].

   But the next two are rather just opposite to the properties of the $SU(5)$ GUT, thus justifying the name Anti-GUT:

3. The various irreducible representations of Weyl fields for the SM group remain irreducible under G. This is the most arbitrary of our assumptions about G. It is motivated by the observation that combining SM irreducible representations into larger unified representations introduces symmetry relations between Yukawa coupling constants, whereas the particle spectrum doess not exhibit any exact degeneracies (except possibly for the case $m_b = m_\tau$). In fact AGUT only gets the naive $SU(5)$ mass predictions as order of magnitude relations: $m_b \approx m_\tau$, $m_s \approx m_\mu$, $m_d \approx m_e$.

4. G is the maximal group satisfying the other 3 postulates. We argued in the previous section that the large number of order of magnitude classes of fermion mass matrix elements indicates the need for a large number of cross product factors in G.

With these four postulates a somewhat cumbersome calculation shows that, modulo permutations of the various irreducible representations in the Standard Model fermion system, we are led to our gauge group $SMG^3 \times U(1)_f$. Furthermore it shows that the SM group is embedded as the diagonal subgroup of SMG^3, as in our AGUT model.

Several of the anomalies involving this $U(1)_f$ are in our solution cancelled by assigning equal and opposite values of the $U(1)_f$ charge to the analogous particles belonging to second and third proto-generations, while the first proto-generation particles have just zero charge [7].

In fact the $U(1)_f$ group does not couple to the left-handed particles and the $U(1)_f$ quantum numbers can be chosen as follows for the proto-states:

$$Q_f(\tau_R) = Q_f(b_R) = Q_f(c_R) = 1 \tag{5.7}$$

$$Q_f(\mu_R) = Q_f(s_R) = Q_f(t_R) = -1 \tag{5.8}$$

Thus the quantum numbers of the quarks and leptons are uniquely determined in the AGUT model. However we do have the freedom of choosing the gauge quantum numbers of the Higgs fields responsible for the breaking the $SMG^3 \times U(1)_f$ group down to the SM gauge group. These quantum numbers are chosen with a view to fitting the fermion mass and mixing angle data [8], as discussed in the next section.

5 Symmetry breaking by Higgs fields

There are obviously many different ways to break down the large group G to the much smaller SMG. However, we can first greatly simplify the situation by assuming that, like the quark and lepton fields, the Higgs fields belong to singlet or fundamental representations of all non-abelian groups. The non-abelian representations are then determined from the $U(1)_i$ weak hypercharge quantum numbers, by imposing the charge quantisation rule for each of the SMG_i groups. So now the four abelian charges, which we express in the form of a charge vector

$$\vec{Q} = \left(\frac{y_1}{2}, \frac{y_2}{2}, \frac{y_3}{2}, Q_f\right)$$

can be used to specify the complete representation of G. The constraint that we must eventually recover the SM group as the diagonal subgroup of the SMG_i groups is equivalent to the constraint that all the Higgs fields (except for the Weinberg-Salam Higgs field which of course finally breaks the SMG) should have charges y_i satisfying:

$$y = y_1 + y_2 + y_3 = 0 \tag{5.9}$$

in order that their SM weak hypercharge y be zero.

We wish to choose the charges of the Weinberg-Salam (WS) Higgs field so that it matches the difference in charges between the left-handed and right-handed physical top quarks. This will ensure that the top quark mass in the SM is not suppressed relative to the WS Higgs field VEV. However, as we remarked in the previous section, it is necessary to associate the physical right-handed top quark field not with the corresponding third proto-generation field t_R but rather with the right-handed field c_R of the second proto-generation. Otherwise we cannot suppress the bottom quark and tau lepton masses. This is because, for the proto-fields, the charge differences between t_L and t_R are the same as between b_L and b_R and also between τ_L and τ_R. So now it is simple to calculate the quantum numbers of the WS Higgs field ϕ_{WS}:

$$\vec{Q}_{\phi_{WS}} = \vec{Q}_{c_R} - \vec{Q}_{t_L} = \left(0, \frac{2}{3}, 0, 1\right) - \left(0, 0, \frac{1}{6}, 0\right) = \left(0, \frac{2}{3}, -\frac{1}{6}, 1\right) \tag{5.10}$$

This means that the WS Higgs field will in fact be coloured under both $SU(3)_2$ and $SU(3)_3$. After breaking the symmetry down to the SM group, we will be left with the usual WS Higgs field of the SM and another scalar which will be an octet of $SU(3)$ and a doublet of $SU(2)$. This should not present any phenomenological problems, provided this scalar doesn't cause symmetry breaking and doesn't have a mass less than the few TeV scale. In particular an octet of $SU(3)$ cannot lead to baryon decay. In our model we take it that what in the standard model is seen as often very small yukawa-couplings to the standard model Higgs field really represent chain Feynmann diagrams composed of propagators with Planck scale heavy particles (fermions) interspaced with the couplings by order of unity yukawa couplings to in our model postulated Higgs fields with names W,T,ξ, and$S$ breaking the AGUT to the Standard Model Group and meaning that the vacuum expectation value is active. The smallness of the effective Yukawa coupling in the Standard Model is taken to be due to the expectation values of W, T, and ξ relative to the masses occuring in the propagators for the Planck scale fermions in the diagrams simulated by the effective Yukawa couplings in the Standard Model.

The quantum numbers of our invented Higgs fields W, T, ξ and S are invented - and it is remarkable that we succeeded so well - so as to make the order of magnitude for the suppressions of the mass matrix elements of the various mass matrices fit to the phenmenological requirements.

After the choice of the quantum numbers for the replacement of the Weinberg Salam Higgs field in our model (5.10) the quantum numbers further needed to be picked out of vacuum in order to give say the mass of the b-quark is denoted by $\vec{b}$ and analogously for the other particles, e.g.:

$$\vec{b} = \vec{Q}_{b_L} - \vec{Q}_{b_R} - \vec{Q}_{WS} \tag{5.11}$$

$$\begin{aligned} \vec{c} &= \vec{Q}_{c_L} - \vec{Q}_{t_R} + \vec{Q}_{WS} && (5.12) \\ \vec{\mu} &= \vec{Q}_{\mu_L} - \vec{Q}_{\mu_R} - \vec{Q}_{WS} && (5.13) \end{aligned}$$

Here we denoted the quantum numbers quarks and leptons as e.g. $\vec{c_L}$ for the left handed components of the proto-charmed quark. Note that $\vec{c}$ has been defined using the t_R proto-field, since we have essentially swapped

the right-handed charm and top quarks. Also the charges of the WS Higgs field are added rather than subtracted for up-type quarks.

Next we attempted to find some Higgs quantum numbers which if postulated to have "small" expectation values compared to the masses of intermediate particles - i.e. denominators in propagators that could go into diagrams and give a fit of the orders of magnitudes. We have the proposal:

$$\vec{Q}_W = \frac{1}{3}(2\vec{b}+\vec{\mu}) = \left(0, -\frac{1}{2}, \frac{1}{2}, -\frac{4}{3}\right) \tag{5.14}$$

$$\vec{Q}_T = \vec{b} - \vec{Q}_W = \left(0, -\frac{1}{6}, \frac{1}{6}, -\frac{2}{3}\right) \tag{5.15}$$

$$\vec{Q}_\xi = \vec{Q}_{d_L} - \vec{Q}_{s_L} = \left(\frac{1}{6}, 0, 0, 0\right) - \left(0, \frac{1}{6}, 0, 0\right) = \left(\frac{1}{6}, -\frac{1}{6}, 0, 0\right) \tag{5.16}$$

From the well-known Fritzsch relation [9] $V_{us} \simeq \sqrt{\frac{m_d}{m_s}}$, it is suggested that the two off-diagonal mass matrix elements connecting the d-quark and the s-quark be equally big. We achieve this approximately in our model by introducing a special Higgs field S, with quantum numbers equal to the difference between the quantum number differences for these 2 matrix elements in the down quark matrix. Then we postulate that this Higgs field has a VEV of order unity in fundamental units, so that it does not cause any suppression but rather ensures that the two matrix elements get equally suppressed. Henceforth we will consider the VEVs of the new Higgs fields as measured in units of M_F and so we have:

$$< S >= 1 \tag{5.17}$$

$$\begin{aligned} \vec{Q}_S &= [\vec{Q}_{s_L} - \vec{Q}_{d_R}] - [\vec{Q}_{d_L} - \vec{Q}_{s_R}] \\ &= \left[\left(0, \frac{1}{6}, 0, 0\right) - \left(-\frac{1}{3}, 0, 0, 0\right)\right] - \left[\left(\frac{1}{6}, 0, 0, 0\right) - \left(0, -\frac{1}{3}, 0, -1\right)\right] \\ &= \left(\frac{1}{6}, -\frac{1}{6}, 0, -1\right) \end{aligned} \tag{5.18}$$

The existence of a non-suppressing field S means that we cannot control phenomenologically when this S-field is used. Thus the quantum numbers of the other Higgs fields W, T, ξ and ϕ_{WS} given above have only been determined modulo those of the field S.

6 Mass matrices, predictions

We define the mass matrices by considering the mass terms in the SM to be given by:

$$\mathcal{L} = Q_L M_u U_R + Q_L M_d D_R + L_L M_l E_R + \text{h.c.} \tag{5.19}$$

The mass matrices can be expressed in terms of the effective SM Yukawa matrices and the WS Higgs VEV by:

$$M_f = Y_f \frac{< \phi_{WS} >}{\sqrt{2}} \tag{5.20}$$

We can now calculate the suppression factors for all elements in the Yukawa matrices, by expressing the charge differences between the left-handed and right-handed fermions in terms of the charges of the Higgs fields. They are given by products of the small numbers denoting the VEVs in the fundamental units of the fields W, T, ξ and the of order unity VEV of S. In the following matrices we simply write W instead of $< W >$ etc. for the VEVs. With the quantum number choice given above, the resulting matrix elements are—but remember that "random" order unity factors are supposed to multiply all the matrix elements—for the uct-quarks:

$$Y_U \simeq \begin{pmatrix} S^\dagger W^\dagger T^2 (\xi^\dagger)^2 & W^\dagger T^2 \xi & (W^\dagger)^2 T \xi \\ S^\dagger W^\dagger T^2 (\xi^\dagger)^3 & W^\dagger T^2 & (W^\dagger)^2 T \\ S^\dagger (\xi^\dagger)^3 & 1 & W^\dagger T^\dagger \end{pmatrix} \tag{5.21}$$

the dsb-quarks:

$$Y_D \simeq \begin{pmatrix} S W (T^\dagger)^2 \xi^2 & W (T^\dagger)^2 \xi & T^3 \xi \\ S W (T^\dagger)^2 \xi & W (T^\dagger)^2 & T^3 \\ S W^2 (T^\dagger)^4 \xi & W^2 (T^\dagger)^4 & W T \end{pmatrix} \tag{5.22}$$

Table 1: Best fit to conventional experimental data. All masses are running masses at 1 GeV except the top quark mass which is the pole mass.

	Fitted	Experimental
m_u	3.6 MeV	4 MeV
m_d	7.0 MeV	9 MeV
m_e	0.87 MeV	0.5 MeV
m_c	1.02 GeV	1.4 GeV
m_s	400 MeV	200 MeV
m_μ	88 MeV	105 MeV
M_t	192 GeV	180 GeV
m_b	8.3 GeV	6.3 GeV
m_τ	1.27 GeV	1.78 GeV
V_{us}	0.18	0.22
V_{cb}	0.018	0.041
V_{ub}	0.0039	0.0035

and the charged leptons:

$$Y_E \simeq \begin{pmatrix} SW(T^\dagger)^2\xi^2 & W(T^\dagger)^2(\xi^\dagger)^3 & (S^\dagger)^2WT^4\xi^\dagger \\ SW(T^\dagger)^2\xi^5 & W(T^\dagger)^2 & (S^\dagger)^2WT^4\xi^2 \\ S^3W(T^\dagger)^5\xi^3 & (W^\dagger)^2T^4 & WT \end{pmatrix} \tag{5.23}$$

We can now set $S = 1$ and fit the nine quark and lepton masses and three mixing angles, using 3 parameters: W, T and ξ. That really means we have effectively omitted the Higgs field S and replaced the maximal AGUT gauge group $SMG^3 \times U(1)_f$ by the reduced AGUT group $SMG_{12} \times SMG_3 \times U(1)$, which survives the spontaneous breakdown due to S. In order to find the best possible fit we must use some function which measures how good a fit is. Since we are expecting an order of magnitude fit, this function should depend only on the ratios of the fitted masses to the experimentally determined masses. The obvious choice for such a function is:

$$\chi^2 = \sum \left[\ln\left(\frac{m}{m_{\rm exp}}\right)\right]^2 \tag{5.24}$$

where m are the fitted masses and mixing angles and $m_{\rm exp}$ are the corresponding experimental values. The Yukawa matrices are calculated at the fundamental scale which we take to be the Planck scale. We use the first order renormalisation group equations (RGEs) for the SM to calculate the matrices at lower scales.

We cannot simply use the 3 matrices given by eqs. (5.21)–(5.23) to calculate the masses and mixing angles, since only the order of magnitude of the elements is defined. Therefore we calculate statistically, by giving each element a random complex phase and then finding the masses and mixing angles. We repeat this several times and calculate the geometrical mean for each mass and mixing angle. In fact we also vary the magnitude of each element randomly, by multiplying by a factor chosen to be the exponential of a number picked from a Gaussian distribution with mean value 0 and standard deviation 1.

We then vary the 3 free parameters to find the best fit given by the χ^2 function. We get the lowest value of χ^2 for the VEVs:

$$\langle W\rangle = 0.179 \tag{5.25}$$
$$\langle T\rangle = 0.071 \tag{5.26}$$
$$\langle \xi\rangle = 0.099 \tag{5.27}$$

The fitted value of $\langle\xi\rangle$ is approximately a factor of two smaller than the estimate given in eq. above. This is mainly because there are contributions to V_{us} of the same order of magnitude from both Y_U and Y_D. The result [8] of the fit is shown in table 1. This fit has a value of:

$$\chi^2 = 1.87 \tag{5.28}$$

This is equivalent to fitting 9 degrees of freedom (9 masses + 3 mixing angles - 3 Higgs VEVs) to within a factor of $\exp(\sqrt{1.87/9}) \simeq 1.58$ of the experimental value. This is better than would have been expected from an order of magnitude fit.

Table 2: Best fit using alternative light quark masses extracted from lattice QCD. All masses are running masses at 1 GeV except the top quark mass which is the pole mass.

	Fitted	Experimental
m_u	1.9 MeV	1.3 MeV
m_d	3.7 MeV	4.2 MeV
m_e	0.45 MeV	0.5 MeV
m_c	0.53 GeV	1.4 GeV
m_s	327 MeV	85 MeV
m_μ	75 MeV	105 MeV
M_t	192 GeV	180 GeV
m_b	6.4 GeV	6.3 GeV
m_τ	0.98 GeV	1.78 GeV
V_{us}	0.15	0.22
V_{cb}	0.033	0.041
V_{ub}	0.0054	0.0035

We can also fit to different experimental values of the 3 light quark masses by using recent results from lattice QCD, which seem to be consistently lower than the conventional phenomenological values. The best fit in this case [8] is shown in table 2. The values of the Higgs VEVs are:

$$\langle W \rangle = 0.123 \tag{5.29}$$
$$\langle T \rangle = 0.079 \tag{5.30}$$
$$\langle \xi \rangle = 0.077 \tag{5.31}$$

and this fit has a larger value of:

$$\chi^2 = 3.81 \tag{5.32}$$

But even this is good for an order of magnitude fit.

7 Baryogenesis

A very important check of our model is if it can be consistent with the baryogenesis. In our model we have just the SM interactions up to about one or two orders of magnitude under the Planck scale. So we have no way, at the electroweak scale, to produce the baryon number in the universe. There is insufficient CP violation in the SM. Furthermore, even if created, the baryon number would immediately be washed out by sphaleron transitions after the electroweak phase transition. Our only chance to avoid the baryon number being washed out at the electroweak scale is to have a non-zero B-L (i.e. baryon number minus lepton number) produced from the high, i.e. Planck, scale action of the theory. That could then in turn give rise to the baryon number at the electroweak scale. Now in our model the B-L quantum number is broken by an anomaly involving the $U(1)_f$ gauge group. This part of the gauge group in turn is broken by the Higgs field ξ which, in Planck units, is fitted to have an expectation value around 1/10. The anomaly keeps washing out any net $B-L$ that might appear, due to CP-violating forces from the Planck scale physics, until the temperature T of the universe has fallen to $\xi = 1/10$. The $U(1)_f$ gauge particle then disappears from the thermal soup and thus the conservation of B-L sets in. The amount of $B-L$ produced at that time should then be fixed and would essentially make itself felt, at the electroweak scale, by giving rise to an amount of baryon number of the same order of magnitude.

The question now is whether we should expect in our model to have a suffient amount of time reversal symmetry breaking at the epoch when the B-L settles down to be conserved, such that the amount of B-L relative to say the entropy (essentially the amount of 3 degree Kelvin background radiation) becomes large enough to agree with the well-known phenomenological value of the order of 10^{-9} or 10^{-10}. At the time of the order of the Planck scale, when the temparature was also of the order of the of the Planck temparature, even the CP or time reversal violations were of order unity (in Planck units). So at that time there existed particles, say, with order of unity CP-violating decays. However, they had also, in our pure dimensional argument approximation, lifetimes of the order of the Planck scale too. Thus the B-L biased decay products would be dumped at time 1 in Planck units, rather than at time of $B-L$ conservation setting in. In a radiation dominated universe, as we shall assume, the temperature will go like 1/a where a is the radius parameter—the size or scale parameter of the universe. Now the time goes as the square of this size parameter a. Thus the time in Planck

units is given as the temperature to the negative second power

$$t = \frac{0.3}{\sqrt{g} \times T^2} \tag{5.33}$$

where [12] g is the number of degrees of freedom—counted as 1 for bosons but as 7/8 per fermion degrees of freedom—entering into the radiation density. In our model g gets a contribution of $\frac{7}{8} \times 45 \times 2$ from the fermions and 2×37 from the gauge bosons, and in addition there is some contribution from the Higgs particles. So we take g to be of order 100, in our crude estimate of the time t corresponding to the temperature $T = xi = \frac{1}{10}$ in Planck units, when $B - L$ conservation sets in:

$$t \simeq \frac{0.3}{100^{1/2}} \times (\frac{1}{10})^2 = 3 \tag{5.34}$$

By that time we expect of the order of $\exp -3$ particles from the Planck era are still present and able to dump their CP-violating decay products. Of course here the uncertainty of an order of magnitude would be in the exponent, meaning a suppression anywhere between say $\exp 0$ and $\exp -30$ and could thus easily be in agreement with the wanted value of order 5×10^{-10}. This result is encouraging, but clearly a more careful analysis is required.

8 Neutrino oscillations, a problem?

At first it seems a problem to incorporate the neutrino oscillations into our model. The a priori prediction would be that the neutrino masses are predicted so small that they could not be seen with presetn accuracy because see-saw mass of the order of Planck scale combined with further suppression leads to too small neutrino masses. However, by changing the system of Higgses and getting the neutrino overall mass scale a fitted number it has been possible to get a satisfactory scheme involving further Higgs fields making shortcuts in the sense of producing transitions that couls already occur by other Higgs field combinations. The extension to neutrinos is not too attracktive, but tolerable.

9 Connection to MPP

Originally the idea of having SMG^3 type model was developped in connection with Random dynamics ideas of confusion and long time in connection with the idea of requiring many phases to meet (multiple point principle MPP) in order to get predictions for the fine structure constants. This type of calculations predicted even that there be three generations at a time when that was not known experimentally by fitting the fine structure constants !(see e.g.([13]))

10 Conclusion

We have looked at some of the hints in the Standard Model that may be usefull in going beyond and have put forward our own model shown to be to some extend inspired by such features:

We stressed to look for the gauge <u>group</u> rather than just the Lie algebra, which a priori is what is relevant for describing a Yang Mills theory.

We have found surprisingly good fits of masses and mixing angles, and in a related model even of the finestructure constants, in a model in which the gauge group at a bit below the Planck scale is the maximal one transforming the already known fermions around and not having anomalies. Although at first it looked a failure it has turned out that even the baryon number generation in big bang is not excluded from being in agreement with the present model. It must , however, then be done by getting first an B-L contribution made at a time when temeperature was only an order of magnitude under the Planck temperature.

To incorporate neutrino oscillations severe but tolerable modifications of the model are needed.

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Hints from the Standard Model for Particle Masses and Mixing

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Abstract

The standard model taken with a momentum space cut-off may be viewed as an effective low energy theory. The structure of it and its known parameters can give us hints for relations between these parameters and for possible extensions. In the present investigation the Higgs problem will be discussed, effective potentials, the possible connection of the Higgs meson with the heavy top quark and the geometric structure of the quark and lepton mass matrices.

1 Introduction

The standard model is likely to describe the effective interaction at low energy of an underlying more fundamental theory. One may speculate that some parameters which emerge at long distances are insensitive to details of what is going on at much higher scales. For instance, their values may be given by the fix points of renormalization group equations and thus being rather independent of the starting numbers at small distances[1] [1]. Or, these parameters could arise in a bootstrap-type scenario [2].

In this talk I do not want to discuss specific models of this type, but will simply look at the measured parameters of the standard model and at its divergence structure in order to find hints for possible connections between these parameters. I will concentrate on the vacuum expectation value of the Higgs field, which is defined without reference to external particles and their momenta. Thus, its divergence property may be quite different from those of ordinary coupling constants, which can be renormalized using a momentum subtraction scheme.

By taking the standard model as an effective theory, one should use a momentum cut-off. The dependence of measurable quantities on the cut-off reflects the influence of new physics on the low energy domain. The minimization of this influence provides suggestions for the sought relations.

2 The vacuum expectation value of the Higgs field and the invariant Higgs potential

We write the Higgs part of the Lagrangian in the form

$$\begin{aligned} \mathcal{L}_H &= (D_\mu\Phi)^\dagger D_\mu\Phi + \frac{J}{2}\Phi^\dagger\Phi - \frac{\lambda}{2}(\Phi^\dagger\Phi)^2 \\ &\quad + \frac{1}{\sqrt{2}} j\cdot\Phi \quad + \quad \text{Yukawa couplings} \\ \Phi &= \frac{1}{\sqrt{2}}\begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_0 + i\varphi_3 \end{pmatrix}. \end{aligned}$$

The quantity J is taken to be $J = J_0 + J_1$ with $J_0 > 0$ describing the Higgs mass parameter responsible for spontaneous symmetry breaking. J_1 can be viewed as an outside field. It is used for generating a gauge invariant potential and will finally be set to zero. The quantity j determines the field direction of the spontaneous symmetry breaking. It could be caused by a light quark condensate and may be neglected after the occurrence of the symmetry breaking. Accordingly, the tree potential takes the form

$$V_0 = \frac{\lambda}{8}(\sum_i \varphi_i^2)^2 - \frac{J}{4}\sum_i \varphi_i^2 - \frac{1}{2}j\varphi_0 \quad . \tag{6.1}$$

For $J > 0$ the minimum of V_0 occurs for $\varphi_i = \hat{\varphi}_i(J, j)$ with

$$\hat{\varphi}_{1,2,3} = 0, \quad \lambda\hat{\varphi}_0^3 - J\hat{\varphi}_0 = j \quad . \tag{6.2}$$

[1] Even the group structure of the standard model and 4-dimensional space could possibly be selected through such mechanisms [1].

We have to select the real solution of the cubic equation. In the limit $j \to 0$, $\lambda \neq 0$ one gets

$$\begin{aligned}\hat{\varphi}_0(J) &= \sqrt{\frac{J}{\lambda}}, \qquad m_H^2(J) = \frac{\partial^2 V_0}{\partial \varphi_0^2}|_{\varphi=\hat{\varphi}} = J \\ & \qquad m_i^2 = \frac{\partial^2 V_0}{\partial \varphi_i^2}|_{\varphi=\hat{\varphi}} = 0 \qquad i = 1,2,3 \quad .\end{aligned} \tag{6.3}$$

By replacing as usual $\varphi_0(x)$ by

$$\varphi_0(x) = \hat{\varphi}_0(J,j) + H(x) \tag{6.4}$$

the interaction part $\mathcal{L}'_{int}$ of the shifted Higgs Lagrangian allows to evaluate $< H >$. The lowest-order expression is

$$< H >= i \int d^4x < 0|T(H(0), \mathcal{L}'_{int}(x))|0 > . \tag{6.5}$$

The result [3] obtained from (6.5) can be used to write the gauge-invariant vacuum expectation value of the square of the Higgs field $\sigma(J) =< \sum_i \varphi_i^2 >$ for $J > 0$ and $j = 0$ in the form

$$\begin{aligned}\sigma(J) &= \tfrac{J}{\lambda} - 2 < H^2 > +2\tfrac{g^2+g'^2}{4\lambda} < Z_\mu Z^\mu > \\ &\quad +4\tfrac{g^2}{4\lambda} < W_\mu^+ W^{-\mu} > -\tfrac{g_t^2}{\lambda} < \bar{t}t > /m_t \quad .\end{aligned} \tag{6.6}$$

g, g' are the gauge couplings for the vector bosons W and Z, and g_t denotes the Yukawa coupling for the top quark. Fermions of lower mass are neglected, but could easily be added. The "vacuum leaks" $< H^2 >$ $,< Z_\mu Z^\mu >, ...$ could be finite in the full theory with a correspondingly modified vacuum structure. For the effective theory with a particle momentum cut-off chosen to be universal for all propagators (6.6) leads to

$$\begin{aligned}\sigma(J) &= \tfrac{J}{\lambda} + \tfrac{\Lambda^2}{8\pi^2}2 \\ &-\tfrac{\Lambda^2}{8\pi^2}[3 + 3\tfrac{g^2+g'^2}{4\lambda} + 6\tfrac{g^2}{4\lambda} - 12\tfrac{g_t^2}{2\lambda}] \\ &+\tfrac{J}{8\pi^2}\left\{1 + 3\left(\tfrac{g^2+g'^2}{4\lambda}\right)^2 + 6\left(\tfrac{g^2}{4\lambda}\right)^2 - 12\left(\tfrac{g_t^2}{2\lambda}\right)^2\right\}\ln\tfrac{\Lambda^2}{J/\lambda} \\ &-\tfrac{J}{8\pi^2}\left(\ln\lambda + 3\left(\tfrac{g^2+g'^2}{4\lambda}\right)^2\ln\tfrac{g^2+g'^2}{4}\right. \\ &\left.+6\left(\tfrac{g^2}{4\lambda}\right)^2\ln\tfrac{g^2}{4} - 12\left(\tfrac{g_t^2}{2\lambda}\right)^2\ln\tfrac{g_t^2}{2}\right) \quad .\end{aligned} \tag{6.7}$$

The term $\frac{\Lambda^2}{8\pi^2}2$ which represents the free field part of $< \Sigma_i \varphi_i^2 >$ is written separately. It also appears in the case $J < 0$ where one has $\hat{\varphi}_0(J) = 0$ and no spontaneous symmetry breaking. For $J < 0$ one finds to one loop order

$$\sigma(J) = \frac{\Lambda^2}{8\pi^2}2 + \frac{J}{8\pi^2}\ln\frac{2\Lambda^2}{-J} \quad . \tag{6.8}$$

Thus, in our approximation[2], $\sigma(J)$ is discontinuous at $J = 0$, indicating a first-order phase transition with strength proportional to Λ^2:

$$\Delta\sigma = -\frac{\Lambda^2}{8\pi^2}[3 + 3\frac{g^2+g'^2}{4\lambda} + 6\frac{g^2}{4\lambda} - 12\frac{g_t^2}{2\lambda}] \tag{6.9}$$

If we would keep $j \neq 0$, the jump of $\sigma(J)$ would undergo a rapid, but now continuous, change in the region $-j^{2/3} \leq J \leq j^{2/3}$. The right-hand side of (6.9) with its quadratic divergence also shows up in the conventional (non-gauge invariant) Higgs potential $V(\varphi)$. Its appearance constitutes an essential part of the hierarchy problem and necessitates fine-tuned subtractions or the postulate of a cancellation of the special combinations of couplings occurring here. The mass relation for which (6.9) vanishes, is

$$S_{\Lambda^2} = (3m_H^2 + 3m_Z^2 + 6m_W^2 - 12m_t^2)/ < \varphi_0 >^2 = 0 \quad . \tag{6.10}$$

It is known as the Veltman condition [4].

Eq. (6.7) can also be obtained in a more general context. By defining the "free energy" $W(J,j)$ by the logarithm of the partition function with the action according to (6.1), one can obtain σ from

$$\sigma = -4\frac{\partial W(J,j)}{\partial J} \quad . \tag{6.11}$$

[2] At $J = 0$ the one-loop approximation for $\sigma(J)$ is presumably insufficient. However, (6.9) is expected to hold for the change of σ within a larger region around $J = 0$.

The Legendre transformation of $W(J,j)$ with respect to J defines the σ-dependent effective potential [5]

$$V(\sigma,j,J_0) = W(J(\sigma,j),j) + \frac{\sigma}{4}(J(\sigma,j) - J_0) \quad . \tag{6.12}$$

It has an extremum at $\bar{\sigma}$, for which $J(\bar{\sigma},j) = J_0$ where J_0 is the mass parameter in the Higgs potential.

Calculating $W(J,j)$ by the saddle point method up to one-loop order, one gets

$$\begin{array}{c} W(J,j) = \frac{\lambda}{8}\hat{\varphi}_0^4(J,j) - \frac{J}{4}\hat{\varphi}_0^2(J,j) - \frac{j}{2}\hat{\varphi}_0(J,j) \\ +\frac{1}{(4\pi)^2}\frac{1}{2}\sum_p r_p \int_0^{\Lambda^2} dK^2 K^2 \ln\left(1+\frac{m_p^2(J,j)}{K^2}\right) \quad . \end{array} \tag{6.13}$$

The sum is over the particles of the standard model with their J- and j-dependent masses. r_p is a statistical factor (3 for the Z, 6 for the W, -12 for the top, 1 for the Higgs). For $j=0$ the result is gauge-invariant. To consider the dependence of $W(J,j=0)$ on J rather than on j has the additional advantage that for $J>0$ the Goldstone particles remain massless and thus do not contribute. Furthermore, to one-loop order, the potential $V(\sigma,J_0)$ remains real for all values of σ. The derivative of $W(J)$ with respect to J according to (6.11) reproduces eq. (6.7). As long as $\Delta\sigma$ is not very small and Λ of order TeV or larger, $V(J_0,\sigma)$ calculated from (6.11–6.13) changes rapidly in its dependence on J_0 for J_0 near zero (and small j).

For the purpose of renormalization we can add to $W(J,j)$ a polynomial in J up to second order. The linear piece could be used to cancel the quadratic divergence in (6.7). However, it would then reappear in (6.8). Instead, one should normalize σ such that it is zero in the limit of all particle masses going to zero, starting from $J<0$: $\sigma(J\to 0_-, j\to 0) = 0$. To achieve this, we have to subtract the $2\Lambda^2/8\pi^2$ part in (6.7) and (6.8) by replacing $W(J,j)$ in (6.13) by $W(J,j)+\frac{\Lambda^2}{32\pi^2}2J$. A further change of $W(J,j)$ using a subtraction term proportional to J^2 can remove the logarithmic divergence in (6.7) and (6.12). This subtraction can be interpreted as a renormalization of the Higgs coupling constant λ. Again, we do not perform such a complete subtraction since the corresponding term would then appear in (6.8) where it has no physical basis. But we can remove the logarithmic divergence for regions of J where the gauge bosons and the fermions remain massless. Since I am not completely certain about the necessity of this subtraction I will consider two cases i) no subtraction proportional to J^2 and – the more appealing one – ii) a subtraction such that in (6.8) besides the quadratic divergence also the logarithmic divergence is removed. Accordingly, the factor which governs the logarithmic divergence of σ and $V(\sigma,J_0)$ in the region of spontaneous symmetry breaking is (to one loop order and in terms of masses)

$$S_{Log\ \Lambda} = (\zeta m_H^4 + 3m_Z^4 + 6m_W^4 - 12m_t^4)/<\varphi_0>^4 \quad . \tag{6.14}$$

$\zeta=1$ corresponds to no subtraction, while $\zeta=0$ is valid when the subtraction according to ii) is performed. (I do not consider here the conventional non gauge invariant potential $V(J_0,<\varphi_0>)$[6]. It would lead to $\zeta=3/2$).

As a speculation I will now assume a minimum influence of new physics on the standard model. In particular, $\Delta\sigma$ of eq. (6.9) should be independent of Λ^2, i.e. the square bracket in (6.9) should be proportional to $1/\Lambda^2$ or zero. If this is the case, the particle couplings are not independent of each other, but satisfy – at least approximately – the Veltman condition. Here one encounters the problem of the scale (μ) at which the particle couplings should be taken. In particular, the Yukawa coupling of the top quark is sensitive to it. The vacuum expectation value of the unrenormalized Higgs field is scale-invariant. But to take advantage of this fact, higher order calculations and a knowledge of the scale dependence of Λ (Λ may be related to high mass states) would be needed. The natural scale for the couplings occuring in the loop integrals is $\mu\approx\Lambda$. Let us first assume that the cancellation of the coupling terms in (6.9) occurs already at the weak scale of $\approx 250\ GeV$, where the top mass is still big. Then, using for the mass of the top $m_t(m_Z) = 173$ GeV the Higgs mass is predicted to be $m_H(m_Z)\approx 280\ GeV$.

Now we can use this value of the Higgs mass to look at the logarithmic divergence and calculate $S_{Log\ \Lambda}$. Using $\zeta=1$ and again the scale $\mu\approx 250\ GeV$ we find $S_{Log\ \Lambda} = -0.2$. We thus have the surprising result that for a Higgs mass of about 300 GeV the factors responsible for the quadratic and the logarithmic divergence of the Higgs potential $V(\sigma,J_0)$ are both small. Let us then consider the extreme case of strictly vanishing factors in front of Λ^2 and $\log\Lambda$ for the one-loop Higgs potential $V(\sigma,J_0)$. This provides us with two equations which allow a calculation of m_t and m_H in terms of the gauge couplings for W and Z. The result is (for $\mu = 250\ GeV$ and a correspondingly very low cut off value)

$$m_t(m_Z) = 198\ GeV \ , \qquad m_H(m_Z) = 320\ GeV \quad . \tag{6.15}$$

The fact that the value of m_t obtained this way is not far away from the experimental result may be a fortuitous coincidence. But if not, it indicates a very close connection of Higgs and top with a Higgs mass not much different from $2m_t\approx 350$ GeV! On the other hand, the prefered value $\zeta=0$ gives no admissible solution at the very low scale considered so far.

Owing to the quadratic form of the two equations which simultaneously suppress quadratic and logarithmic divergences, another type of solution exists with smaller values for the Higgs mass. For this solution the scale relevant for the particles running in the loops and thus the value of the cut off Λ must be extremly high in order to get a large enough value for the top mass at the weak scale. We take, therefore, μ equal to the Planck mass, determine m_t and m_H and apply the two loop renormalization group equations to get their values at the weak scale. This implies, of course, that physics beyond the standard model, which could influence the standard model couplings, can occur only near and above the Planck scale. The calculation using $\alpha_s(m_Z) = 0.12$ and $\zeta = 1$ gives

$$m_t(m_Z) = 169\ GeV\ ,\ m_H(m_Z) = 140\ GeV \tag{6.16}$$

and $\zeta = 0$

$$m_t(m_Z) = 168\ GeV\ ,\ m_H(m_Z) = 137\ GeV\ \ . \tag{6.17}$$

Both solution do not differ much since λ at the Planck scale is found to be small (but not zero). I prefer the solution with $\zeta = 0$ because of the good behaviour (no divergence) of σ in the broken as well as in the unbroken phase. Furthermore, $S_{Log\Lambda}$ can be expressed in terms of the β-function of $\frac{J_0^2}{8\lambda}$, i.e. the β-function of the zero order expression of $W(J)$ [6]. The numbers obtained in (6.16) and (6.17) differ little from the result obtained by Bennett, Nielsen and Froggatt in the framework of their anti grand unification model. This model requires $\lambda(m_{Planck}) = 0$, (not far away from $\lambda(m_{Planck}) = 0.04$ obtained here) and the vanishing of $S_{Log\Lambda}$.

We have seen, that the possibility still exists that the particles of the standard model have adjusted their couplings such as to stabilize their masses. If this is so, the hierarchy problem is no more a problem of protecting the mass of the spin zero Higgs particle compared to the masses of the spin 1/2 particles which are protected by chiral symmetry.

Clearly, as long as higher-order calculations of $V(\sigma, J_0)$ are not available, the scale dependence of the couplings entering the expression for $\sigma(J_0)$ brings in large uncertainties. So far it is only a hope that the one-loop result is not entirely misleading us. In higher-loop calculations the quadratic divergence is not well defined. One needs a lattice regularization or a regularization by supersymmetry at some high scale. In the latter case the quadratic divergence is only an effective one up to the high scale which could again be the Planck scale.

3 Masses and Mixings of Quarks and Leptons

The possible intimate relation between Higgs and top discussed in the previous paragraph suggests also a dominant role of the top for the structure of the quark and lepton mass matrices. The masses of the lighter particles can then be expected to be related to the top mass by powers of a small constant [7, 8]. Let us look at the quark and charged lepton masses at the common scale m_Z in the $\overline{MS}$ scheme (in GeV) [9]

$$\begin{array}{lll} m_t = 173 \pm 6 & m_b = 2.84 \pm 0.10 & m_\tau = 1.78 \\ m_c = 0.58 \pm 0.06 & m_s = (70 \pm 14)10^{-3} & m_\mu = 106 \times 10^{-3} \\ m_u = (2.0 \pm 0.5)10^{-3} & m_d = (3.6 \pm 0.8)10^{-3} & m_e = 0.51 \times 10^{-3} \end{array} \tag{6.1}$$

We take as the small parameter $\epsilon \simeq \sqrt{\frac{m_c}{m_t}} = 0.058 \pm 0.004$. Then, the rule $m_t : m_c : m_n = 1 : \epsilon^2 : \epsilon^4$ may be supposed to hold up to $O(\epsilon)$ corrections. Similarly, the ratio of down quark masses and the ratio of charged lepton masses are taken to be simple rational numbers times integer powers of ϵ. It is then plausible to use a corresponding geometric structure also for the off-diagonal elements of the mass matrices. Here, I will not go into details since the paper containing these suggestions is published [8]. I will only quote the results: The up-quark matrix can be taken real and symmetric. For the down-quark and the charged lepton matrices the simplest possible textures are used for which the 3rd generation decouples from the first and second. The first and second generation can mix by a complex entry. Taking this mixing coefficient of the light generations purely imaginary, one obtains – for fixed ϵ – a maximal CP-violation. It manifests itself by making the unitarity triangle to be a right-handed one with the unitarity angle $\gamma \simeq 90^o$. Besides the top mass and ϵ there are only two additional parameters: the beauty to top and the τ to beauty mass ratios. With these parameters one gets the following, so far quite successful, numbers (in GeV):

$$\begin{array}{lll} m_t = 174 & m_b = 2.8 & m_\tau = 1.77 \\ m_c = 0.58 & m_s = 84 \times 10^{-3} & m_\mu = 103 \times 10^{-3} \\ m_u = 1.95 \times 10^{-3} & m_d = 4.2 \times 10^{-3} & m_e = 0.52 \times 10^{-3} \end{array} \tag{6.2}$$

$$|V_{us}| = 0.216,\ |V_{cb}| = 0.041,\ |V_{ub}| = 0.0034,\ |V_{td}| = 0.0094, \tag{6.3}$$

and $\alpha = 70^0,\ \beta = 20^o,\ \gamma = 90^o$.

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Composite Models and Susy

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1 Introduction

The idea that the standard model of strong and electroweak interactions itself is an effective theory of another, more fundamental interaction and quarks, leptons and bosons are composites of more fundamental fields is almost as old as the standard model itself. Since quarks carry flavor and color from a purely spectroscopical point of view, one can think of them as being glued together from entities having flavor only ("flavons") and others having color only ("chromons"). To get a similar picture for leptons, Pati and Salam,(1) who invented this idea as early as 1974, viewed the lepton number as a fourth color. A new QCD-like gauge interaction ("metacolor," "hypercolor") binds the constituents (preons) together and its residual Van-der-Waals-type interactions are the interactions of the standard model.

But there are other reasons of taking the existence of strong fundamental gauge interactions seriously. They come from the — up to now — unexplored Higgs-sector of the standard model. Actually, the only job of the Higgs doublet in the standard model is to provide symmetry breaking. This requires a certain form of the Higgs potential, so the ground state of the system is no longer invariant under $SU(2)_L \times U(1)$. Although there is nothing wrong with this mechanism, we have not observed such a pattern for elementary boson fields, but what we have observed is spontaneous symmetry breaking in QCD, where the global chiral symmetry of the quarks is broken by condensates $\langle \bar{q}_L q_R \rangle \neq 0$ as effect of the strong color force. In fact, technicolor theories(2) have been invented only for this purpose.

Once familiar with the idea, one will address the whole flavor problem to composite models. In fact, any underlying strong interaction theory must yield the Yukawa couplings, the condensate values and therefore the masses of fermions as a result rather than as fundamental parameters. It is therefore a challenging task for any model builder to provide at least a gross understanding of the mass hierarchies, as well as to give an argument how many families we should expect.

Some composite models(3,4) also address problems usually associated with Grand Unified Theories, like the existence and incorporation of additional baryon-number (or even B-L)-violating interactions. Here the experimental evidence is the existence of the baryon asymmetry in the universe, a measurement to be taken seriously by any particle physicist. This, however, means having the compositeness scale very high $\Lambda > 10^{11}$ GeV, which usually makes it difficult but not impossible(5) to make low energy predictions.

2 The Size of Composite Quarks

If a particle is composite, its most important parameter is its size. The inverse size of a particle shows up as a scale parameter in all couplings with dimension, like magnetic moments. The easiest way to define the size of a fermion(3) is by the strong four-fermi-interaction

$$\mathcal{L}_{ST} = \frac{k^2}{M^2} \cdot \bar{\psi}_\alpha \psi_\alpha \cdot \bar{\psi}_\alpha \psi_\alpha \ . \tag{7.1}$$

For a strong interaction, we expect $\frac{k^2}{4\pi} \sim 0(1)$ and define

$$\Lambda_\alpha^2 = \frac{4\pi}{k^2} \cdot M^2 \ . \tag{7.2}$$

For instance, from low-energy proton-proton scattering, we find for the nucleon

$$\Lambda_p \sim m_\rho \sim 700 \text{ MeV} \ . \tag{7.3}$$

In a preonic theory, all the inverse sizes of the composites should be related to a fundamental scale Λ. This scale is usually defined by the value of the condensates

$$\langle \bar{f}_L^a f_R^b \rangle \simeq A^{ab} \Lambda^3 \tag{7.4}$$

where the flavor matrix A^{ab} is assumed to be $O(1)$ unless there exists a peculiar suppression mechanism, as we will discuss below.

If we assume that the composite fermion ψ_α acquires its mass through dynamical breaking of chiral symmetry, and if in addition the fermion f is a constituent of ψ, we find the relation[6]

$$\Lambda_\alpha = \frac{1}{r_\alpha} = k\left(\frac{\Lambda^3}{m_\alpha}\right)^{\frac{1}{2}} . \tag{7.5}$$

This relation is well known for QCD, where $\Lambda = \Lambda_{QCD} \simeq 250$ MeV, $m_\alpha \equiv m_p \simeq 1$ GeV, $k \sim g_{QCD} = g_{\rho NN} = 5.5$. We find $\Lambda_p \sim 700$ MeV in agreement with equ.(3).

Now let us impose equ. (5) on a preonic-type theory, yielding composite quarks. In this case the condensate in (4) is the one relevant for $SU(2)_L \times U(1)$-breaking, ψ_α is a quark, and f is one of its constituents. Since, however, the condensates as well as the masses form a matrix in flavor space, we can only give a statement about the heaviest eigenvalue. This yields,

$$(\Lambda_q)_H = k\left[\Lambda_{HC}^3/m(q_H)\right]^{1/2} . \tag{7.6}$$

Assuming $m(q_H) = m_{top} = 175$ GeV, $k^2 \simeq 10$, $\Lambda_{HC} \simeq (\sqrt{2}G_F)^{-1/2} \simeq 150$ GeV from electroweak symmetry breaking, we arrive at $(\Lambda_a)_{HC} < 1{,}6$ TeV. We would like to stress the point that this result is quite general, as long as the *same* constituents are used making up the quark as well as the condensate and as long as there is no special suppression mechanism.

3 Constraints on Preon Dynamics

If we take the more radical point of view and assume *all* particles are composite, even leptons, gluons, and photons, the compositeness scale is presumably very high, $\Lambda = \Lambda_M \simeq 10^{11}$ GeV. Such a theory must have a spectrum of massless composites, and therefore also a residual chiral symmetry to protect fermion masses and a residual gauge symmetry to prevent vector meson masses. Let us outline a few general theorems which are important in such a case.

Assume we start with a preonic theory (QPD) based on fundamental fermions (with or without fundamental bosons), which has an invariance group G_M(local) $\times$ G_{fc} (global or local). At a certain scale Λ_M the local G_M interactions become strong and form bound states (composites), which are G_M singlets. In addition, part of G_{fc} is spontaneously broken by condensates $\langle \bar{f}f\rangle_o \neq 0$, yielding an effective low energy symmetry group $G_{LE} \subset G_{fc}$ for the composites. This effective symmetry group is local, if the composite vector mesons carrying the quantum numbers of G_{LE} are born massless at Λ_M, otherwise it is global. Can G_{LE} contain the familiar $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group?

Before going to certain details, we should remark the following: Certainly any effective theory among composite objects is nonrenormalizable, because one can write down arbitrarily high dimensional operators in terms of the fundamental fields. However, in terms of composite fields, d-dimensional operators are suppressed by a scale factor $\Lambda_M^{-(d-4)}$ $(d > 4)$, and their contribution is of order $(m/\Lambda_M)^{d-4}$, where m is the mass of the composites. Therefore, if we restrict ourselves to the mass zero composites (*i.e.* $m/\Lambda_M << 1$), the remaining low-dimensional operators have to be renormalizable and therefore are assumed to be a gauge theory (with certain Yukawa couplings).

One has further to distinguish whether G_M is vectorlike or chiral. In the latter case, the preons have to be in a complex representation R alone without being accompanied by another set of preons belonging to $\bar{R}$. In the first case, one has the appealing feature that the underlying vectorlike gauge theory is responsible for the formation of chiral composites and parity violation. This is a challenging task, in particular since for such QCD-like theories a number of constraints and no-go-theorems have been worked out:

(i) The Weingarten,[7] Nussinov,[8] and Witten[9] mass inequalities require, among other things, that the lightest composite fermion be heavier than the light composite boson:

$$m_F > (N/2)m_B \tag{7.7}$$

(assuming $G_M = SU(N)$);

(ii) The Weinberg-Witten theorem[10] shows that in a vectorlike theory with a Lorentz-covariant conserved current (energy-momentum tensor), no composite particle with spin $j > 1/2$ $(j > 1)$ with mass zero and corresponding charge unequal zero can be formed.

(iii) The Vafa-Witten theorem[11] shows that vectorlike *global* symmetries $U_{L+R}(N)$ are not broken by condensates, and massless bound states do not form from massive constituents.

(iv) Contrary to this, Weingarten[(7)] has shown , using lattice field theory calculations, that global chiral symmetries $U_L(N) \times U_R(N)$ are broken at the composite level.

(v) Furthermore, even in chiral QPD, one has to fulfill 't Hooft's anomaly-matching conditions[(12)] which state that the Adler-Bell-Jackiw anomalies[(13)] associated with background gauge fields of G_{fc} have to be the same at the preon as at the composite level. (It should be noted that G_M , as a gauge theory, has to be anomaly-free anyway in order to be consistent.) This puts certain severe restrictions on the preon representation of G_M in chiral QPD.

We see that the mass inequalities[(7−9)] and the Vafa-Witten constraint prohibit the appearance of a residual $SU(2)_L$ protecting the masses of composite quarks and leptons (without having lighter composite bosons) in a vectorlike QPD. Fortunately, these constraints can be avoided in a *supersymmetric* QPD for the following reasons:

(i) Both the Weingarten inequality and the Vafa-Witten theorem use the positivity of the fermion determinant in QCD-type theories in their proof. However, in supersymmetric QCD-theories, there exist Yukawa-interactions between gauginos, matter fermions and spin-zero matter bosons which destroy the positivity.[(14)]

(ii) The proof of the breakdown of chiral symmetry in lattice QCD also ignores the presence of Yukawa interactions.

(iii) In a local supersymmetric Yang-Mills theory there is no Lorentz-covariant conserved current and energy-momentum tensor.

(iv) The anomaly matching conditions are easily fulfilled in a rather economical way, if quarks and leptons are constructed as fermion-boson composites (as is the case in supersymmetric theories).[(15)]

4 SUSY-Quantumpreondynamics: A specific model

The necessity for supersymmetry at the preon level opens two new aspects. First, one is led to the intriguing possibility that the supersymmetric QPD itself is the result of a supergravity or superstring theory at a higher scale, presumably the Planck scale. Secondly, one has to answer how the supersymmetry is broken. The most promising way is to assume the breakdown of local supersymmetry through gaugino condensates[(16)] formed by the G_M-gauge force itself at Λ_M . This in turn implies the appearance of a vacuum expectation value for the auxiliary fields and therefore a breakdown of global supersymmetry. There exists however, certain restrictions on global SUSY-breaking, depending on the representation and mass of your fundamental preon fields.[(17)] For such cases, one finds that the flavor matrix A^{ab} in front of the scale parameter Λ^3 has to be damped by powers of $\Lambda_M/M_{P\ell}$. In the following, we give an example of a SUSY-composite model of this type.

The model under consideration[18,19] is based on a set of massless chiral superfields, each belonging to the fundamental representation of the metacolor gauge symmetry $SU(N)$. The superfields carry also flavor-color quantum numbers according to the gauge group $SU(4)_c \times SU(2)_L \times SU(2)_R$:

$$\Phi^{a,\sigma}_{\pm} = (\varphi, \psi, F)^{a,\sigma}_{L,R} \tag{7.8}$$

where σ denotes the metacolor index, a denotes flavor-color indices up/down and red, yellow, blue and lilac for the lepton color.

It is assumed that the metacolor force becomes strong and confining at a scale $\Lambda_M \simeq 10^{11}$ GeV, with the following effects:

- Three light chiral families[(19)] of composite quarks and leptons $\left(q^i_{L,R}\right)_{i=1,2,3}$ and two vector-like families $Q_{L,R}$ and $Q'_{L,R}$, coupling vectorially to W_L's and W_R's, are formed:

$$\begin{aligned} q_L &= \Phi^a_+(\Phi^r_-)^* \simeq (2,1,4), \\ q_R &= \Phi^a_-(\Phi^r_+)^* \simeq (1,2,4), \\ Q_{L,R} &= \Pi_\pm(\Phi^a_+(\Phi^r_+)^*) \simeq (2,1,4), \\ Q'_{L,R} &= \Pi_\pm(\Phi^a_-(\Phi^r_-)^*) \simeq (1,2,4) \end{aligned} \tag{7.9}$$

where $\Pi_\pm$ are the projection operators onto left/right chiral superfields;

- Supersymmetry-breaking condensates are formed; they include the metagaugino condensate $\langle\vec{\lambda}\cdot\vec{\lambda}\rangle$ and the matter fermion-condensates $\langle\bar{\psi}^a\psi^a\rangle$. Noting that, within the class of models under consideration, the index theorem prohibits a dynamical breaking of supersymmetry in the absence of gravity[20], so the formation of these condensates must need the collaboration between the metacolor force and gravity. As a result, each of these condensates is expected to be damped by one power of $(\Lambda_M/M_{P\ell}) \simeq 10^{-8}$ relative to Λ_M [17]:

$$\begin{aligned} \langle\vec{\lambda}\cdot\vec{\lambda}\rangle &= \kappa_\lambda \Lambda_M^3 \,(\Lambda_M/M_{P\ell})\,, \\ \langle\bar{\psi}^a\psi^a\rangle &= \kappa_{\psi_a} \Lambda_M^3 \,(\Lambda_M/M_{P\ell}) \end{aligned} \qquad (7.10)$$

  Here, the indices a are running over color and flavor quantum numbers. The condensates $\langle\bar{\psi}^a\psi^a\rangle$, break not only SUSY but also the electroweak symmetry $SU(2)_L\times U(1)_Y$ therefore giving mass to the electroweak gauge bosons. The coefficients κ_λ and κ_{ψ_a}, apriori, are expected to be of order unity within a factor of ten (say), although κ_λ is expected to be bigger than κ_ψ's, typically by factors of 3 to 10, because the ψ's are in the fundamental and the λ's are in the adjoint representation of the metacolor group.

- Furthermore, supersymmetry-preserving condensates, which however break the gauge group $SU(4)_c\times SU(2)_L\times SU(2)_R$ to the low-energy gauge group $SU(3)_c\times SU(2)_L\times U(1)_Y$ are assumed to form as well. They provide a large superheavy Majorana mass to the right-handed neutrinos and may play an interesting role in the discussion of inflationary models [21].

Now, the vector-families $Q_{L,R}$ and $Q'_{L,R}$ acquire relatively heavy masses through the metagaugino condensate $\langle\vec{\lambda}\cdot\vec{\lambda}\rangle$ of order $\kappa_\lambda\Lambda_M\,(\Lambda_M/M_{P\ell}) \sim 1$ TeV which are independent of flavor and color. But the chiral families $q^i_{L,R}$ acquire masses primarily through their mixings with the vector-like families $Q_{L,R}$ and $Q'_{L,R}$ which are induced by $\langle\bar{\psi}^a\psi^a\rangle$. This is because the direct mass-terms cannot be induced through two-body condensates. Thus, ignoring QCD corrections and higher order condensates for a moment, the Dirac-mass matrices of all four types - i.e., up, down, charged lepton, and neutrino - have the form:

$$M^{(o)} = \begin{array}{c} \\ \overline{q^i_R} \\ \overline{Q_R} \\ \overline{Q'_R} \end{array} \begin{array}{c} \begin{array}{ccc} q^i_L & Q_L & Q'_L \end{array} \\ \begin{pmatrix} O & X\kappa_f & Y\kappa_c \\ Y'^\dagger\kappa_c & \kappa_\lambda & O \\ X'^\dagger\kappa_f & O & \kappa_\lambda \end{pmatrix} \end{array} \quad . \qquad (7.11)$$

Here, the index i runs over three families, f,c denotes flavor- or color-type condensate, and the quantitites X, Y, X' and Y' are column matrices in the family-space and have their origin in the detailled vertex structure of the corresponding preonic diagrams. They are expected to be numbers of order $\simeq 1$. As a result, the Dirac mass-matrices of all four types have a natural see-saw structure.

Naturally, the key ingredient in this prediction is the seesaw mechanism, provided by the existence of the two set of vector–like quarks. The prediction of the masses, coupling constants and decay rates of these quarks and their verification by experiments is therefore a crucial test of the model itself. For this reason, detailled predictions for the discovery of the new heavy vectorlike Quarks have been worked out. Let us shortly repeat the results:

- We expect four heavy quarks U_1, U_2, D_1, D_2 as well as four leptons E_1, E_2, N_1, N_2 whose electroweak properties are given by Q, Q' including their mixing;

- Fitting the six parameters of the mass matrix and allowing for about 10% corrections due to electroweak final state interactions, one gets

$$\begin{array}{ll} U_1 \simeq 1814\ GeV & N_1 \simeq 765\ GeV \\ D_1 \simeq 1787\ GeV & E_1 \simeq 763\ GeV \\ U_2 \simeq 1504\ GeV & N_2 \simeq 581\ GeV \\ D_2 \simeq 1465\ GeV & E_2 \simeq 667\ GeV \end{array} \qquad (7.12)$$

In summary, two vector–like families, not more not less, with one coupling vectorially to W_L's and the other to W_R's (before mass–mixing), with masses of order $\simeq 1TeV$, constitute a *hall–mark* and a crucial prediction of the SUSY preon model [18,19] under consideration. There does not seem to be any other model including superstring–inspired models of elementary quarks and leptons which have a good reason to predict two such complete vector–like families with masses in the TeV range.

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Discussions

Are spins and charges unified? How can one otherwise understand the connection between the handedness (the spin) and the weak charge?

Anamarija[1] and Norma

This question seems to Norma the essential open question of the Standard electroweak model. The answer to this question should show the way beyond the Standard model.

1 Introduction

In Norma's talk the algebras and subalgebras of two kinds of generators of the Lorentz transformations, defining in 14-dimensional Grassmann space[2] the group $SO(1,13)$, both the linear differential operators, were presented and some of their representations were discussed. According to two kinds of generators defined in the linear vector space, spanned over Grassmann coordinate space, there are also two kinds of representations: we call them spinorial and vectorial representations, respectively. We choose Grassmann odd polynomials to describe spinorial and Grassmann even polynomials to describe vectorial kind of vectors.

Since the group $SO(1,d-1)$ contains for $d = 14(+1)$ as subgroups the groups $SO(1,3)$, needed to describe spins of fermions and bosons, as well as $U(1), SU(2)$ and $SU(3)$, needed to describe the Yang-Mills charges of fermions and bosons, **the spin and the Yang - Mills charges of either fermions or bosons are in the presented approach unified**. Since spins and charges are described by the representations of the generators of the Lorentz transformations of either fermionic or of bosonic character, it means that **fermionic states must belong to the spinorial representations with respect to the groups, describing charges, while bosonic states must belong to the vectorial representations with respect to the groups, describing charges**.

Among representations of the proposed approach are the ones, needed to describe the quarks, the leptons and the gauge bosons, which appear in the Standard electroweak model[5, 4]. We find left handed spinors, $SU(3)$ triplets and $SU(2)$ doublets with $U(1)$ charge $\frac{1}{6}$ and right handed spinors, $SU(3)$ triplets and $SU(2)$ singlets with $U(1)$ charge $\frac{2}{3}$ and $-\frac{1}{3}$, which describe quarks. We find left handed spinors, $SU(3)$ singlets and $SU(2)$ doublets with $U(1)$ charge $-\frac{1}{2}$ and right handed spinors, $SU(3)$ singlets and $SU(2)$ singlets with $U(1)$ charge 0 or -1, which describe leptons. We find also the corresponding representations for anti quarks and anti leptons. We find the four vectors, $SU(3)$ triplets and $SU(2)$ singlets with $U(1)$ charge 0, describing gluons and $SU(3)$ singlets and $SU(2)$ triplets with $U(1)$ charge 0, describing massless weak bosons and $SU(3)$ singlets and $SU(2)$ singlets with $U(1)$ charge 0, describing a $U(1)$ gauge field. One can find[4] for vectorial case besides octets and singlets of $SU(3)$ also triplets and besides triplets and singlets of $SU(2)$ also doublets. These representations have an odd Grassmann character in $SO(10)$ and the correspondingly even or odd Grassmann character in the rest of space. Accordingly the Higgs's boson of this model[3] appears as a scalar, which is a $SU(3)$ singlet and $SU(2)$ doublet, with an odd Grassmann character in $SO(1,3)$ and $SU(2)$ part of the Grassmann space. Since the four dimensional subspace of the Grassmann space, above which the group $SO(1,3)$ is defined, has $2^4 = 16$ basic functions, **the approach predicts four** rather than three **families of quarks and leptons**[4], provided that this symmetry manifests already on the level of quarks and leptons.

However, the supersymmetric partners of the gauge bosons, required by the supersymmetric extension of the Standard Electroweak Model, can in the proposed theory exist only as constituent particles. In this contribution we demonstrate that Mankoč approach, defining all the internal degrees of freedom in an unique way in Grassmann space and accordingly unifying spins and charges, offers a possible explanation why the postulate of the Standard model that only left handed fermions carry the weak charge occurs, or equivalently why the weak interaction breaks left-right symmetry.

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[2]In the talk of Norma one additional dimension to 14 dimensions was proposed, needed to properly define γ^a matrices, in the contribution to the discussions written by Norma and Holger, another definition of the γ^a matrices was proposed, which does not need the additional dimension. The algebra of the group $SO(1,13)$ does not depend on the additional dimension.

[3]Higgs could appear also as a constituent field!

[4] See also the contribution to the discussions written by Norma and Holger

2 Subgroups of SO(1,7), SO(1,9) and SO(1,13) and representations

We first define the operators, which in Grassmann space define vectorial and spinorial representations. We start with

$$p^{\theta a} := i\overrightarrow{\partial}^{\theta}_{a}, \;\; \tilde{a}^a := i(p^{\theta a} - i\theta^a), \;\; \tilde{\tilde{a}}^a := -(p^{\theta a} + i\theta^a). \tag{1.1}$$

We find

$$\{p^{\theta a}, p^{\theta b}\} = 0 = \{\theta^a, \theta^b\}, \;\; \{p^{\theta a}, \theta^b\} = i\eta^{ab}, \tag{1.2}$$

$$\{\tilde{a}^a, \tilde{a}^b\} = 2\eta^{ab} = \{\tilde{\tilde{a}}^a, \tilde{\tilde{a}}^b\}, \;\; \{\tilde{a}^a, \tilde{\tilde{a}}^b\} = 0, \;\; \tilde{\gamma}^a := i\tilde{a}^0\tilde{a}^a. \tag{1.3}$$

Then we define two kinds of binomials. The first kind is made of operators forming the Heisenberg odd algebra

$$S^{ab} := (\theta^a p^{\theta b} - \theta^b p^{\theta a}), \tag{1.4}$$

the second kind of operators forming the Clifford algebra

$$\tilde{S}^{ab} = \frac{i}{4}[\tilde{a}^a, \tilde{a}^b] = \frac{i}{4}[\tilde{\gamma}^a, \tilde{\gamma}^b], \;\; \text{with } [A,B] = AB - BA. \tag{1.5}$$

Either S^{ab} or $\tilde{S}^{ab}$ fulfill the Lorentz algebra.
We shall write M^{ab} for either S^{ab} or $\tilde{S}^{ab}$. We shall use the space of coordinates θ^m, $m \in \{0,1,2,3\}$ to describe spin degrees of freedom and $h \in \{5,6,..,d\}$ to describe charges.

2.1 $SO(1,3)$

We define the representations of the group $SO(1,3)$ through two $SU(2)$ subgroups in the standard way

$$N_i^{\pm} := \frac{1}{2}(\frac{1}{2}\epsilon_{ijk}M^{ik} \pm iM^{0i}), \;\; [N_i^{\pm}, N_j^{\pm}] = i\epsilon_{ijk}N_k^{\pm}, \;\; [N_i^{\pm}, N_j^{\mp}] = 0. \tag{1.6}$$

We shall present in Table I the representations of only the spinorial type of generators. Taking into account the expressions for the operators $\tilde{S}^{mn}$ one easily finds the eigenvectors of the Casimirs $\sum_i(\tilde{N}_i^{\pm})^2$ and $\tilde{N}_3^{\pm}$ as the polynomials of θ^m.

a	i	$<\theta\|\varphi_i^a>$	$\tilde{S}_3$	$\tilde{K}_3$	$\tilde{\Gamma}$	family
1	1	$\frac{1}{2}(\tilde{a}^1 - i\tilde{a}^2)(\tilde{a}^0 - \tilde{a}^3)$	$-\frac{1}{2}$	$\frac{i}{2}$	-1	I
1	2	$-\frac{1}{2}(1 + i\tilde{a}^1\tilde{a}^2)(1 - \tilde{a}^0\tilde{a}^3)$	$\frac{1}{2}$	$-\frac{i}{2}$	-1	
2	1	$\frac{1}{2}(\tilde{a}^1 - i\tilde{a}^2)(\tilde{a}^0 + \tilde{a}^3)$	$-\frac{1}{2}$	$-\frac{i}{2}$	1	
2	2	$-\frac{1}{2}(1 + i\tilde{a}^1\tilde{a}^2)(1 + \tilde{a}^0\tilde{a}^3)$	$\frac{1}{2}$	$\frac{i}{2}$	1	
3	1	$\frac{1}{2}(\tilde{a}^1 - i\tilde{a}^2)(1 - \tilde{a}^0\tilde{a}^3)$	$-\frac{1}{2}$	$-\frac{i}{2}$	1	II
3	2	$-\frac{1}{2}(1 + i\tilde{a}^1\tilde{a}^2)(\tilde{a}^0 - \tilde{a}^3)$	$\frac{1}{2}$	$\frac{i}{2}$	1	
4	1	$\frac{1}{2}(\tilde{a}^1 - i\tilde{a}^2)(1 + \tilde{a}^0\tilde{a}^3)$	$-\frac{1}{2}$	$\frac{i}{2}$	-1	
4	2	$-\frac{1}{2}(1 + i\tilde{a}^1\tilde{a}^2)(\tilde{a}^0 + \tilde{a}^3)$	$\frac{1}{2}$	$-\frac{i}{2}$	-1	
5	1	$\frac{1}{2}(1 - i\tilde{a}^1\tilde{a}^2)(\tilde{a}^0 - \tilde{a}^3)$	$-\frac{1}{2}$	$\frac{i}{2}$	-1	III
5	2	$-\frac{1}{2}(\tilde{a}^1 + i\tilde{a}^2)(1 - \tilde{a}^0\tilde{a}^3)$	$\frac{1}{2}$	$-\frac{i}{2}$	-1	
6	1	$\frac{1}{2}(1 - i\tilde{a}^1\tilde{a}^2)(\tilde{a}^0 + \tilde{a}^3)$	$-\frac{1}{2}$	$-\frac{i}{2}$	1	
6	2	$-\frac{1}{2}(\tilde{a}^1 + i\tilde{a}^2)(1 + \tilde{a}^0\tilde{a}^3)$	$\frac{1}{2}$	$\frac{i}{2}$	1	
7	1	$\frac{1}{2}(! - i\tilde{a}^1\tilde{a}^2)(1 - \tilde{a}^0\tilde{a}^3)$	$-\frac{1}{2}$	$-\frac{i}{2}$	1	IV
7	2	$-\frac{1}{2}(\tilde{a}^1 + i\tilde{a}^2)(\tilde{a}^0 - \tilde{a}^3)$	$\frac{1}{2}$	$\frac{i}{2}$	1	
8	1	$\frac{1}{2}(1 - i\tilde{a}^1\tilde{a}^2)(1 + \tilde{a}^0\tilde{a}^3)$	$-\frac{1}{2}$	$\frac{i}{2}$	-1	
8	2	$-\frac{1}{2}(\tilde{a}^1 + i\tilde{a}^2)(\tilde{a}^0 + \tilde{a}^3)$	$\frac{1}{2}$	$-\frac{i}{2}$	-1	

Table I: The polynomials of θ^m, representing the four times two Dirac bispinors, are written. For each state the eigenvalues of $\tilde{S}_3 := \tilde{S}^{1,2}, \tilde{K}_3, \Gamma := i\tilde{a}^0\tilde{a}^1\tilde{a}^2\tilde{a}^3$ are written. The Roman numerals tell the possible family number. We use the relation $\tilde{a}^a|0>= \theta^a$.
One can check that $\tilde{\gamma}^0$ transforms the two bispinors of each family into one another. One also finds that the operator with an odd Grassmann character $\tilde{a}^m$ transforms the first family into the second and the third into the fourth, while $\tilde{\tilde{a}}^0$ transforms the first family into the third and the second into the fourth. One also finds that complex conjugation transforms the first family into the fourth and the second into the third.

2.2 $SO(4)$

We use the Grassmann coordinates $\theta^5, \theta^6, \theta^7$ and θ^8 to describe the weak and $U(1)$ charge.
We define the generators of the two groups

$$\tau^{11} := \frac{1}{2}(M^{58} - M^{67}),\ \ \tau^{12} := \frac{1}{2}(M^{57} + M^{68}),\ \ \tau^{13} := \frac{1}{2}(M^{56} - M^{78}), \tag{1.7}$$

while

$$\tau^{2w} := \frac{1}{2}(M^{56} + M^{78}). \tag{1.8}$$

One finds

$$[\tau^{Ai}, \tau Bj] = \delta^{AB}\delta^{A1} i\epsilon_{ijk}\tau^{Ak}, \tag{1.9}$$

with $A \in (1,2),\ \ i \in (1,..,n_A),\ \ n_A = 3$ for $SU(2)$ and 1 for $U(1)$. We present in Table II the eigenstates of the Casimirs $\sum_i (\tau^{1i})^2$ andτ^{2w} and of τ^{13} for spinorial degrees of freedom.

2.3 $SO(1,7)$

We assume that the representations of the group $SO(1,7)$ are the direct product of representations of the subgroups $SO(1,3)$ and $SO(4)$. Taking into account the results of the last two subsections, one finds that the operator $\tilde{S}^{mh}, m \in (0,1,2,3), h \in (5,6,7,8)$ transforms the left handed $SU(2)$ doublets into right handed $SU(2)$ singlets. Accordingly one finds in a $SO(1,7)$ multiplet left handed doublets and right handed singlets. One can, of course, also find multiplets with left handed singlets and right handed doublets, since the left-right symmetry is not broken (yet).

Table II: The eigenstates of the τ^{13}, τ^{2w} are presented. We find two doublets and four singlets of an even Grassmann character and two doublets and four singlets of an odd Grassmann character. One sees that complex conjugation transforms one doublet of either odd or even Grassmann character into another of the same Grassmann character changing the sign of the value of τ^{13}, while it transforms one singlet into another singlet of the same Grassmann character and of the opposite value of τ^{2w}. One can check that $\tilde{a}^h,\ \ h \in (5,6,7,8)$, transforms the doublets of an even Grassmann character into singlets of an odd Grassmann character.

a	i	$< \theta\vert\varphi^a{}_i >$	$\tilde{\tau}^{13}$	$\tilde{\tau}^{2w}$	Grass. character
1	1	$\frac{1}{2}(1 - i\tilde{a}^5\tilde{a}^6)(1 + i\tilde{a}^7\tilde{a}^8)$	$-\frac{1}{2}$	0	even
1	2	$-\frac{1}{2}(\tilde{a}^5 + i\tilde{a}^6)(\tilde{a}^7 - i\tilde{a}^8)$	$\frac{1}{2}$	0	
2	1	$\frac{1}{2}(1 + i\tilde{a}^5\tilde{a}^6)(1 - i\tilde{a}^7\tilde{a}^8)$	$\frac{1}{2}$	0	
2	2	$-\frac{1}{2}(\tilde{a}^5 - i\tilde{a}^6)(\tilde{a}^7 + i\tilde{a}^8)$	$-\frac{1}{2}$	0	
3	1	$\frac{1}{2}(1 + i\tilde{a}^5\tilde{a}^6)(1 + i\tilde{a}^7\tilde{a}^8)$	0	$\frac{1}{2}$	
4	1	$\frac{1}{2}(\tilde{a}^5 + i\tilde{a}^6)(\tilde{a}^7 + i\tilde{a}^8)$	0	$\frac{1}{2}$	
5	1	$\frac{1}{2}(1 - i\tilde{a}^5\tilde{a}^6)(1 - i\tilde{a}^7\tilde{a}^8)$	0	$-\frac{1}{2}$	
6	1	$\frac{1}{2}(\tilde{a}^5 - i\tilde{a}^6)(\tilde{a}^7 - i\tilde{a}^8)$	0	$-\frac{1}{2}$	
7	1	$\frac{1}{2}(1 + i\tilde{a}^5\tilde{a}^6)(\tilde{a}^7 - i\tilde{a}^8)$	$\frac{1}{2}$	0	odd
7	2	$-\frac{1}{2}(\tilde{a}^5 - i\tilde{a}^6)(1 + \tilde{a}^7\tilde{a}^8)$	$-\frac{1}{2}$	0	
8	1	$\frac{1}{2}(1 - i\tilde{a}^5\tilde{a}^6)(\tilde{a}^7 + i\tilde{a}^8)$	$-\frac{1}{2}$	0	
8	2	$-\frac{1}{2}(\tilde{a}^5 + i\tilde{a}^6)(1 - i\tilde{a}^7\tilde{a}^8)$	$\frac{1}{2}$	0	
9	1	$\frac{1}{2}(1 - i\tilde{a}^5\tilde{a}^6)(\tilde{a}^7 - i\tilde{a}^8)$	0	$-\frac{1}{2}$	
10	1	$\frac{1}{2}(\tilde{a}^5 + i\tilde{a}^6)(1 + \tilde{a}^7\tilde{a}^8)$	0	$\frac{1}{2}$	
11	1	$\frac{1}{2}(1 + i\tilde{a}^5\tilde{a}^6)(\tilde{a}^7 + i\tilde{a}^8)$	0	$\frac{1}{2}$	
12	1	$\frac{1}{2}(\tilde{a}^5 - i\tilde{a}^6)(1 - \tilde{a}^7\tilde{a}^8)$	0	$-\frac{1}{2}$	

2.4 $SO(6)$

We use the Grassmann coordinates $\theta^9, \theta^{10}, \theta^{11}, \theta^{12}, \theta^{13}$ and θ^{14} to describe the colour and $U(1)$ charge.
We define the generators of the two groups as follows

$$\tau^{3\,1} := \frac{1}{2}(M^{9\,12} - M^{10\,11}),\ \ \tau^{3\,2} := \frac{1}{2}(M^{9\,11} + M^{10\,12}),\ \ \tau^{3\,3} := \frac{1}{2}(M^{9\,10} - M^{11\,12}),$$

$$\tau^{3\,4} := \frac{1}{2}(M^{9\,14} - M^{10\,13}),\;\; \tau^{3\,5} := \frac{1}{2}(M^{9\,13} + M^{10\,14}),\;\; \tau^{3\,6} := \frac{1}{2}(M^{11\,14} - M^{12\,13}),$$

$$\tau^{3\,7} := \frac{1}{2}(M^{11\,13} + M^{12\,14}),\;\; \tau^{3\,8} := \frac{1}{2\sqrt{3}}(M^{9\,10} + M^{11\,12} - 2M^{13\,14}), \tag{1.10}$$

while

$$\tau^{2c} := -\frac{1}{3}(M^{9\,10} + M^{11\,12} + M^{13\,14}). \tag{1.11}$$

One finds

$$[\tau^{Ai}, \tau^{Bj}] = \delta^{AB}\;\; \delta^{A3}\;\; i f^{A}_{ijk}\;\; \tau^{Ak}, \tag{1.12}$$

with $A \in (1,2)$, $i \in (1,..,n_A)$, $n_A = 8$ for $SU(3)$ and 1 for$U(1)$. The constants f^A_{ijk} are the structure constants of the group $SU(3)$.
We can again find the eigenstates of $\sum_i(\tau^{3i})^2$, $\tau^{3\,3}$, $\tau^{3\,8}$ and $\tau^{2\,c}$ and present them as polynomials of θ^k, $k \in (9,10,11,12,13,14)$. We find triplets, anti triplets, singlets and anti singlets[5, 4]. The operator $\tilde{a}^k$, $k \in (9,10,11,12,13,14)$ transforms Grassmann even triplets into Grassmann odd anti triplets and singlets.

2.5 $SO(1,9)$

We assume the representations of the group $SO(1,9)$ to be the direct product of the representations of the group $SO(1,3)$ and the group $SO(6)$. Accordingly one finds that left handed $SU(3)$ triplets and right handed $SU(3)$ anti triplets and singlets appear in the same $SO(1,9)$ multiplet. We also have right handed triplets and left handed anti triplets and anti singlets in the another multiplet. One has to find out what would the break of the left-right symmetry bring.

2.6 $SO(1,13)$

We may assume that the representations of the group $SO(1,13)$ are the direct product of the representations of the groups $SO(1,3)$, $SU(4)$ and $SU(6)$ and that the $U(1)$ charge is the sum of the τ^{2w} and τ^{2c} charges:$Y := \tau^{2w} + \tau^{2w}$.
In Table III we present the quantum numbers of fermions, which appear in the proposed approach as members of **one multiplet of the group $SO(1,13)$,** if members of the same multiplet are found by the application of the operators $\tilde{S}^{ab}, a,b \in \{0,1,.,8\}$ on any starting representation. One finds the four bispinors (rather than one), which differ among themselves with respect to discrete symmetries of the Lorentz subgroup $SO(1,3)$ and which may represent four (rather than three) families of quarks and leptons.

					SU(2) doublets				SU(2) singlets			
family	I	II	III	IV	$\tilde\tau^{13}$	$\tilde\tau^{2w}+\tilde\tau^{2c}$	$\tilde{Q}$	$\tilde\Gamma^{(4)}$	$\tilde\tau^{13}$	$\tilde\tau^{2w}+\tilde\tau^{2c}$	$\tilde{Q}$	$\tilde\Gamma^{(4)}$
SU(3) triplets												
$\tilde{\tau^{3\,3}} = (\ \frac{1}{2}, -\frac{1}{2}, 0\ )$	u_1	u_2	u_3	u_4	1/2	1/6	2/3	± 1	0	2/3	2/3	∓ 1
$\tilde\tau^{3\,8} = (\ \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}, -\frac{1}{\sqrt{3}}\ )$	d_1	d_2	d_3	d_4	-1/2	1/6	-1/3	± 1	0	-1/3	-1/3	∓ 1
SU(3) singlets												
$\tau^{3\,3} = 0$	ν_1	ν_2	ν_3	ν_4	1/2	-1/2	0	± 1	0	0	0	∓ 1
$\tau^{3\,8} = 0$	e_1	e_2	e_3	e_4	-1/2	-1/2	1	± 1	0	-1	-1	∓ 1

Table III: Quantum numbers of fermions for chiral multiplets of the group $SO(1,13)$, which represent four families of quarks and leptons. Quantum numbers of the corresponding antifermions have the opposite signs. We introduce $\tilde{Q} = \tilde\tau^{13} + \tilde\tau^{2w} + \tilde\tau^{2c}$. To the column of $SU(2)$ doublets with $\tilde\Gamma = \pm 1$, the column of $SU(2)$ singlets with $\tilde\Gamma = \mp 1$, correspond.

3 Conclusions and what have we learned

We have presented the approach, which unifying spins and charges, connects lefthanded $SU(2)$ doublets and right handed $SU(2)$ singlets into the same multiplet, manifesting that handedness and weak charges of fermions are connected in a way, needed in the Standard model. The operators $\tilde{S}^{mh}, m \in (0,1,2,3), h \in (5,6..)$, which cause such transformations, appear in the equations of motions of the approach (See Eq.(5.2) of Norma's talk) in terms, which behave like Yukawa couplings.
The generators of the Lorentz transformations in Grasssmann space, defining the Yang - Mills charges, commute with the generators of the Lorentz transformations in the four dimensional subspace in accordance with the

Coleman - Mandula theorem [2] as well as with it extension for the supersymmetric case [3] as long as the group $SO(1,13)$ manifests as the product of the subgroups $SO(1,3) \times SU(3) \times SU(2) \times U(1)$, which only is true for low enough energies.

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Why do we have parity violation?

Holger and Colin

1 Introduction

Why do we have parity violation, or why is the weak charge dependent on handedness?

The short answer to this question is that we need at least some of the charges to be different for the observed right-handed and left-handed fermion states—i.e. handedness dependent or chiral—for the purpose of mass protection. That is to say: in the philosophy that the particles we "see"—those we can afford to produce and measure—are very light (essentially massless) from the supposed fundamental scale point of view and, thus, they need a mechanism for being exceptionally light so that we have a chance to "see' them. This mechanism should suggestively be that any pair of right and left (Weyl) components should have at least one gauge quantum number different between them, so that any mass term is forbidden by gauge invariance.

Really we should rather ask why is parity conserved in the electromagnetic and strong interactions. Our philosophy would be that a priori there is no reason why these symmetries should be there at all, and it is the presence of the symmetries (rather than their breaking) that needs an explanation. This is the philosophy of what we call random dynamics, which really means: all that is not forbidden occurs. It is very natural since really to know a symmetry exists is much more informative than to know it not to be there. So a priori one should rather say that, if there is no reason for them, we should not expect symmetries to be present.

In the case of the question of whether the electroweak charges on the Weyl components of the quark and lepton fields should be the same for the two handednesses, right and left, we can say that, since the Weyl fields transform under Lorentz transformations without mixing into each other (i.e. they transform into themselves only), we should consider each Weyl field as essentially corresponding to a completely separate particle. As separate particles we expect them to a priori have completely different charges. You might of course object that when the particle has a mass, so that we are talking about a Dirac particle, there is a connection between the left and the right Weyl component fields. However in the Standard Model it is well-known that the masses come about as an effect of the Higgs field vacuum expectation value. So, before the effect of the Higgs mechanism, the fermions are massless and there is no association of the various Weyl field with each other a priori.

Thus the question that really deserves and needs an answer is rather why there is parity conservation for the strong and elctromagnetic interactions, in the sense that the electromagnetic and colour charges are the same on the right and the left components of the same Dirac particle. In addition the fact that the right-handed components are singlets, while the left-handed components are doublets, under the weak SU(2) gauge group needs an explanation. So, assuming the existence of the Standard Model gauge group, we now derive the Standard Model fermion representations on the basis of a few simple assumptions.

2 Starting assumptions

2.1 The assumptions to derive Standard Model representations of fermions

(a) As the starting point for the derivation of the Standard Model representations, we shall assume the gauge <u>group</u> and not only the gauge Lie algebra of the Standard Model to be $S(U(2) \times (U(3))$.

(b) Further we shall make the assumption that the representations—realised by the Weyl fermions—of this group are "small". More specifically we assume that the weak hypercharge charge $y/2$ is at most unity numerically, and that only the trivial and the lowest fundamental (defining) representations of the nonabelian groups SU(2) and SU(3) are used.

(c) Further we assume mass protection, i.e. we say that all particles for which a mass could be made, without the Higgs field being used, would be so heavy that we should not count them as observable particles.

(d) In our argument we shall also use the requirement that there shall be no gauge nor mixed anomalies. This is needed since otherwise there would be a breaking of the gauge symmetry.

These assumptions are of course known to be true in the Standard Model. Indeed they are rather suggestive regularities of the Standard Model, if one is looking for inspiration to go beyond the Standard Model. You could say that it might not be so difficult to find some rather general argumentation for why representations should be "small" in some way—not exactly how small perhaps.

2.2 Slightly reduced assumptions for parity in strong and electromagnetic interactions

¿From the assumptions stated in the foregoing subsection we can indeed derive the fermion representations of the whole Standard Model and, thus, also the fact that there is parity conservation in electromagnetic and strong interactions. However, if we replace the requirement $|y/2| \leq 1$ by the slightly modified assumption that the electric charge $Q = y/2 + I_{W3}$ (where I_{W3} is the third component of the weak isospin) has numerical value less than or equal to unity for all the Weyl fermion representations, the mass protection assumption is not needed for this parity derivation. The point of course is that the mass protection is performed by the weak interaction, and the electromagnetic and colour quantum numbers do not provide any mass protection themselves–they cannot with parity symmetry.

In other words, for the illustrational derivation of parity conservation in strong and electromagnetic interactions alone, we assume:

(a) Either the gauge <u>group</u> $U(3)$ for strong and electromagnetic interactions, or the total gauge group $S(U(2) \otimes U(3))$ as above.

(b) The "small" representations in the form $|Q| \leq 1$ and $\underline{a} \leq \underline{3}$

(c) and then of course that there should still be no anomalies.

3 Derivation of parity for QCD and electromagnetism

The program of our proof of parity for strong and electromagnetic interactions consists in showing that the Weyl fields must have quantum number combinations that will be paired into Dirac fields, so that parity in the electromagnetic and strong interactions gets preserved.

What we have to show is that there are always equally many Weyl field species with a given electric and colour charge combination and the opposite. In this way we could then say that at least the possibility is there for combining these Weyl fields into Dirac fields, so that the electric and colour charges on the right and the left Weyl components become the same. We should of course have in mind that, in four dimensions, one can consider the right handed Weyl components as represented by their CP-conjugates so to speak, meaning a corresponding set of left-handed fields with the opposite charges. So we actually need only discuss the left-handed components, just letting them represent the right-handed ones too as antiparticles.

Now for anomaly calculations it is easily seen that left-handed Weyl fields in conjugate representations give just equal and opposite contributions to the various anomalies. Thus we can only hope to say from anomaly considerations something about the number of species in one representation minus the number in the conjugate one. We should therefore introduce names for these differences:

We let the symbol $N_{(y/2,\underline{a},I_W)}$ denote the number of left-handed Weyl species with the weak hypercharge y/2, the colour representation $\underline{a}$ and the weak isospin I_W minus the number of species with the opposite (conjugate) quantum numbers. But, in the present section, we ignore the weak isospin and use $N_{(Q,\underline{a})}$ to mean the difference of the number of Weyl-species with electric charge Q and color representation $\underline{a}$ and the number of Weyl-species with the conjugate quantum numbers.

The requirement of the smallness of the representations means that $N_{(Q,\underline{a})}$ is zero unless

$$|Q| \leq 1 \tag{2.1}$$

$$|\underline{a}| \leq |\underline{3}| \tag{2.2}$$

Obviously by our definition $N_{(Q=0,\underline{1})} = n - n = 0$.

What we have to show to get parity conservation for these interactions is that

$$N_{(Q,\underline{a})} = 0. \tag{2.3}$$

for all the quantum number combinations $(Q, \underline{a})$.

The requirements of small representations and of the gauge <u>group</u> being $U(3)$ leaves only the three differences of species numbers $N_{(Q=1,\underline{1})}$, $N_{(Q=2/3,\underline{3})}$, $N_{(Q=-1/3,\underline{3})}$ non-zero.

Now the anomalies in four dimensions come from triangle diagrams with external gauge fields for the gauge anomalies and with two gravitons and one gauge particle assigned in the case of the mixed anomaly. In order to get rid of the anomalies, so as to avoid breaking the gauge symmetry say, we must require that the relevant triangle diagrams have cancellations between the contributions coming from the different Weyl field species, the latter circling around the triangle. The only mixed anomaly diagram, not already vanishing for other reasons,

is a triangle with Weyl particles circling around it having two gravitons attached and the photon at the third vertex. The cancellation required to get rid of this the mixed anomaly becomes

$$N_{(Q=1,\underline{1})} + \frac{2}{3} \times 3N_{(Q=2/3,\underline{3})} + (-\frac{1}{3}) \times 3N_{(Q=-1/3,\underline{3})} = 0 \tag{2.4}$$

To ensure no gauge anomaly there are three triangle diagrams that must have a cancellation: one with three external gluons which gives

$$N_{(Q=2/3,\underline{3})} + N_{(Q=-1/3,\underline{3})} = 0, \tag{2.5}$$

one with one photon and two gluons attached, which gives

$$2/3 \times N_{(Q=2/3,\underline{3})} + (-1/3) \times N_{(Q=-1/3,\underline{3})} = 0 \tag{2.6}$$

and finally one with three photons attached which gives

$$N_{(Q=1,\underline{1})} + \left(\frac{2}{3}\right)^3 \times 3N_{(Q=2/3,\underline{3})} + \left(-\frac{1}{3}\right)^3 \times 3N_{(Q=-1/3,\underline{3})} = 0 \tag{2.7}$$

We have here got four linear equations for three unknowns, so it is no wonder that they lead to all the differences being zero. That then means to every Weyl representation there is the possibility of finding just one with the opposite (conjugate) representation. This vanishing of the differences is sufficient to give parity conservation, provided possible mass mechanisms do not violate the gauge symmetries for colour and electromagnetism. It means that one may directly construct a parity operator, by diagonalizing a perhaps present U(3) gauge invariant mass mechanism and letting it map the right to the corresponding left mass eigenstate and opposite.

4 Deriving all standard model fermion representations

Using a very similar technique, but now within all the four assumptions stated in the subsection 2.1, we can show the fermion representations to be those of the Standard Model with some as yet not determined number of generations.

For this purpose the assumption about small representations can be taken to mean that $N_{(y/2,\underline{a},I_W)}$ is zero unless

$$|y/2| \leq 1 \tag{2.8}$$

$$|\underline{a}| \leq |\underline{3}| \tag{2.9}$$

$$|I_W| \leq 1/2. \tag{2.10}$$

Really it means that we assume zero species for the representations not fullfilling this and thus of course the same for the differences $N_{(y/2,\underline{a},I_W)}$. The requirement of the gauge <u>group</u> being $S(U(2) \times U(3))$ means that the species numbers are zero unless the congruence

$$y/2 + t/3 + d/2 = 0 \qquad (\text{mod } 1) \tag{2.11}$$

is fullfilled, where t is triality and d is duality.

Obviously by our definition $N_{(y/2=0,\underline{1},I_W=0)} = n - n = 0$.

The small representation and the gauge <u>group</u> requirements now allow six $N_{(y/2,\underline{a},I_W)}$'s to be nonzero a priori, namely one for each of the allowed numerical values of y/2 which runs from 1/6, in steps of 1/6, to 1.

As above we use the cancellation criteria for the anomalies, meaning the cancellation of triangle diagrams: This time the mixed anomaly cancellation diagram has two gravitons and one weak hypercharge coupling and it gives

$$\begin{aligned} &\frac{1}{6} \times 6N_{(y/2=1/6,\underline{3},I_W=1/2)} + \frac{1}{3} \times 3N_{(y/2=1/3,\underline{\bar{3}},I_W=0)} \\ +&\frac{1}{2} \times 2N_{(y/2=1/2,\underline{1},I_W=1/2)} + \frac{2}{3} \times 3N_{(y/2=2/3,\underline{3},I_W=0)} \\ &+\frac{5}{6} \times 6N_{(y/2=5/6,\underline{\bar{3}},I_W=1/2)} + N_{(y/2=1,\underline{1},I_W=0)} \quad = \quad 0 \end{aligned} \tag{2.12}$$

The no gauge anomaly triangle diagrams consist of one with three external gluons, while the one with three external W's is trivially zero and does not count, then there are two diagrams with respectively two gluons and

two W's and one weak hypercharge coupling, and finally there is one diagram with all three attached gauge particles being the abelian one (coupling to weak hypercharge). The conditions become:

$$\begin{aligned} 2N_{(y/2=1/6,\underline{3},I_W=1/2)} - N_{(y/2=1/3,\underline{\bar{3}},I_W=0)} & \\ +N_{(y/2=2/3,\underline{3},I_W=0)} - 2N_{(y/2=5/6,\underline{\bar{3}},I_W=1/2)} &= 0 \end{aligned} \tag{2.13}$$

$$\begin{aligned} \frac{1}{6} \times 2N_{(y/2=1/6,\underline{3},I_W=1/2)} + \frac{1}{3} \times N_{(y/2=1/3,\underline{\bar{3}},I_W=0)} & \\ +\frac{2}{3} \times N_{(y/2=2/3,\underline{3},I_W=0)} + \frac{5}{6} \times 2N_{(y/2=5/6,\underline{\bar{3}},I_W=1/2)} &= 0 \end{aligned} \tag{2.14}$$

$$\begin{aligned} \frac{1}{6} \times 3N_{(y/2=1/6,\underline{3},I_W=1/2)} + \frac{1}{2} \times N_{(y/2=1/2,\underline{1},I_W=1/2)} & \\ +\frac{5}{6} \times 3N_{(y/2=5/6,\underline{\bar{3}},I_W=1/2)} &= 0 \end{aligned} \tag{2.15}$$

$$\begin{aligned} \left(\frac{1}{6}\right)^3 \times 6N_{(y/2=1/6,\underline{3},I_W=1/2)} + \left(\frac{1}{3}\right)^3 \times 3N_{(y/2=1/3,\underline{\bar{3}},I_W=0)} & \\ +\left(\frac{1}{2}\right)^3 \times 2N_{(y/2=1/2,\underline{1},I_W=1/2)} + \left(\frac{2}{3}\right)^3 \times 3N_{(y/2=2/3,\underline{3},I_W=0)} & \\ +\left(\frac{5}{6}\right)^3 \times 6N_{(y/2=5/6,\underline{\bar{3}},I_W=1/2)} + N_{(y/2=1,\underline{1},I_W=0)} &= 0 \end{aligned} \tag{2.16}$$

Here we have got 5 equations linear in the N's of which there were 6. Thus it is not surprising that there is, up to the unavoidable scaling by a common factor of all the unknowns—the N's—just one solution. This must, however, be that of the Standard Model since the latter satisfies the anomaly cancellation conditions. The scaling factor corresponds to the generation number we could say. So far we have only shown that the N's are as in the Standard Model. We need now to use the assumption about mass protection to deduce that we cannot have both representations—i.e. a representation and its conjugate—associated with a given N present. That implies first that the cases of N's that are zero imply that there will be no Weyl fermions at all associated with those quantum numbers—there will be no vector fermions. Also for the cases of nonzero N's only one of the two associated representations will exist, depending on the sign of the N in question. With this conclusion we almost truly derived the Standard Model fermion representations; there are however still two ambiguities: 1) the generation number can be any integer, 2) we could have the opposite signs for the N's which would correspond to a model that is, so to speak, a parity reflected version of the Standard Model.

Since we have now derived the whole representation system for the fermions in the Standard Model, we did not really need the exercise of deriving parity for the electromagnetic and colour interactions separately; we got it all at once after all, assuming though—as is needed—the Higgs mechanism.

5 Conclusion

We have shown that, from the four requirements above, it is possible to argue for the whole system of the Weyl fermion representations of the Standard Model. So if one can just argue for these assumptions in some model beyond the Standard Model one will have the fermion system for free.

Concerning the question of whether the charges depend on handedness, we saw that for the colour and electric charges no such dependence is allowed, by just using the smallness of electric charge and colour representation plus the no anomaly conditions. Concerning the question of why there is a dependence—namely for the weak charges—we saw that it was the mass protection requirement that enforced it. In fact each of the differences N had to be a difference between zero and another number, because the mass protection would not allow two sets of Weyl fields counted as left-handed having opposite (conjugate) quantum numbers. They would namely combine to get a huge mass according to the philosophy of mass protection. Thus indeed the charges must, in one way or another, be different for the right and left handnesses.

This really means that we take the point of view that the fundamental scale, or the next level in fundamentality, has so huge a characteristic energy or mass scale that all the particles we know must in first approximation be arranged to be massless, i.e. they must be mass protected.

6 Acknowledgement

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The Number of Families in a Scenario with Right-Handed Neutrinos

Astri[4]

In the Standard Model, the number of light neutrino flavours, $< N_\nu >$, is defined by the invisible Z-width, i.e.

$$\Gamma_{inv}(\mathrm{Z} \to \nu'\mathrm{s}) = \Gamma_0 < \mathrm{N}_\nu > \tag{3.1}$$

where Γ_0 is the standard width for a massless neutrino pair, $\Gamma_0 = G_F M_Z^3/12\pi\sqrt{2}$, and $< N_\nu >$ equals the number of left-handed lepton doublets, i.e. the number of families.

In a scenario where (two or more) right-handed neutrinos are included in the Standard Model, we can however draw no definite conclusion about the number of families from the invisible Z-width.

In order to see this, consider the neutral current term (in the physical basis) in an extended Standard Model scenario, with two right-handed neutrinos and n families,

$$\mathcal{L}_{NC} = \frac{g}{2\cos\theta_W} \bar{N}\gamma^\mu \mathbf{\Omega} N Z_\mu \tag{3.2}$$

Here N contains the neutrino fields,

$$N = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ . \\ . \\ . \\ \nu_{nL} \\ 0 \\ 0 \end{pmatrix}$$

and $\mathbf{\Omega}$ is the $(n+2)$x$(n+2)$ matrix

$$\mathbf{\Omega} = \mathbf{U} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \mathbf{U}^\dagger$$

where U is the $(n+2)$x$(n+2)$ neutrino mixing matrix.
The matrix $\mathbf{\Omega}$ is just a unitary transformation of the matrix $diag(1, 1, ..., 1, 0, 0)$. The trace of $\mathbf{\Omega}$ is thus $tr(\mathbf{\Omega}) = n$, and since $\mathbf{\Omega}\mathbf{\Omega}^\dagger = \mathbf{\Omega}^2 = \mathbf{\Omega}$,

$$\sum_{j,k=1}^{n} |\mathbf{\Omega}_{jk}|^2 = tr(\mathbf{\Omega}\mathbf{\Omega}^\dagger) = tr(\mathbf{\Omega}) = n.$$

The neutral current coupling coefficients thus satisfy

$$\sum_{j,k=1}^{n} |\mathbf{\Omega}_{jk}|^2 = n, \tag{3.3}$$

where the right-hand side is just the number of left-handed leptonic doublets, i.e the number of families.

¿From equation (3.2) we have that

$$\Gamma(\mathrm{Z} \to \nu'\mathrm{s}) = \Gamma_0 \sum_{\mathrm{i,j=1}}^{\mathrm{n}} \mathrm{X_{ij}} |\Omega_{\mathrm{ij}}|^2, \tag{3.4}$$

where the X_{ij} are the phase space and matrix element suppression factors due to the nonvanishing neutrino masses, bounded by unity. In a scenario with right-handed neutrinos present, the invisible width therefore satifies the inequality

$$\Gamma(\mathrm{Z} \to \nu'\mathrm{s}) \leq \mathrm{n}\Gamma_0, \tag{3.5}$$

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and no decisive conclusion can thus be drawn about the number of families.

Does this mean that provided there are right-handed neutrinos, data are compatible with more than three families?

What does our model tell about three versus four families? In our specific scenario, the generic neutrino mass matrix is of the form

$$\mathcal{M} = \begin{pmatrix} \mathbf{0} & \mathbf{A} \\ \tilde{\mathbf{A}} & \mathbf{M} \end{pmatrix}, \tag{3.6}$$

and in the case with three families and two right-handed neutrinos, the matrix $\mathbf{A}$ is a 2x3 matrix, and the matrix $\tilde{\mathbf{A}}$ is its transpose, i.e.

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & 0 & a_1 & b_1 \\ 0 & 0 & 0 & a_2 & b_2 \\ 0 & 0 & 0 & a_3 & b_3 \\ a_1 & a_2 & a_3 & M_1 & 0 \\ b_1 & b_2 & b_3 & 0 & M_2 \end{pmatrix} \tag{3.7}$$

In this scheme, with the introduction of certain constraints, the mass spectrum consists of three massless and two massive eigenstates, and the invisible Z-width takes the form

$$\Gamma(\mathrm{Z} \to \nu + \bar{\nu}) = \Gamma_0[1 + 1 + \frac{1}{\mathrm{s}_{2\gamma}^4 \mathrm{c}_{2\theta}^4}(\mathrm{c}_{2\gamma}^4 \mathrm{s}_{2\theta}^4 + 2\mathrm{S}^2 \mathrm{c}_{2\gamma}^2 \mathrm{s}_{2\theta}^2 (\mathrm{c}_\gamma^2 \mathrm{X}_{34} + \mathrm{s}_\gamma^2 \mathrm{X}_{35}) +$$
$$+ S^4(c_\gamma^4 X_{44} + s_\gamma^4 X_{55} + 2c_\gamma^2 s_\gamma^2 X_{45}))] \tag{3.8}$$

where $S = \sqrt{c_{2\theta}^2 - c_{2\gamma}^2}$ and $c_{2\gamma} = \cos 2\gamma$, etc. The angles γ and θ are defined by $M_1/M_2 = -\tan^2\theta$ and $\lambda_+/\lambda_- = \tan^2\gamma$, where $\lambda_\pm$ are the non-vanishing neutrino masses, and constrained by $\tan\gamma \neq 0$, $\tan\gamma \neq 1$, $\tan\theta \neq 0$, $\tan\theta \neq 1$, and $\tan\gamma \neq \tan\theta$. Since the factors X_{ij} are bounded by unity, $\Gamma(\mathrm{Z} \to \nu + \bar{\nu}) \leq 3\Gamma_0$, in agreement with data.

A scheme with four families can be modeled on the case with three families by letting $\mathbf{A}$ be a 2x4 matrix, whereby the mass matrix becomes

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & 0 & 0 & a_1 & b_1 \\ 0 & 0 & 0 & 0 & a_2 & b_2 \\ 0 & 0 & 0 & 0 & a_3 & b_3 \\ 0 & 0 & 0 & 0 & a_4 & b_4 \\ a_1 & a_2 & a_3 & a_4 & M_1 & 0 \\ b_1 & b_2 & b_3 & b_4 & 0 & M_2 \end{pmatrix} \tag{3.9}$$

With a mass spectrum with four zero and two non-vanishing neutrino masses, the corresponding invisible Z-width is

$$\Gamma(\mathrm{Z} \to \nu + \bar{\nu}) = \Gamma_0[1 + 1 + 1 + \frac{1}{\mathrm{s}_{2\gamma}^4 \mathrm{c}_{2\theta}^4}(\mathrm{c}_{2\gamma}^4 \mathrm{s}_{2\theta}^4 + 2\mathrm{S}^2 \mathrm{c}_{2\gamma}^2 \mathrm{s}_{2\theta}^2 (\mathrm{c}_\gamma^2 \mathrm{X}_{45} + \mathrm{s}_\gamma^2 \mathrm{X}_{46}) +$$
$$+ S^4(c_\gamma^4 X_{55} + s_\gamma^4 X_{66} + 2c_\gamma^2 s_\gamma^2 X_{56}))], \tag{3.10}$$

where the definitions and constraints are the same as above. The suppression factors are given by

$$X_{ij} = \frac{\sqrt{\lambda(M_{\mathrm{Z}}^2, m_i^2, m_j^2)}}{M_{\mathrm{Z}}^2 A_{ij}} \tag{3.11}$$

where λ is given by $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + bc + ac)$, and A_{ij} include the mass dependence of the matrix elements.

For small enough $\tan 2\theta / \tan 2\gamma$, as well as S and X_{ij}'s, $\Gamma(\mathrm{Z} \to \nu + \bar{\nu})/\Gamma_0$ is compatible with the experimental value of the number of neutrinos, $< N_\nu >_{exp} = 2.991 \pm 0.016$ (LEP Electroweak Working Group, CERN/PPE/95-172). The actual values of the angles and the other parameters of course depend on the (unknown) neutrino masses and mixings.

So in this model with right-handed neutrinos, with suitable neutrino masses and mixing angles, four families are in principle not excluded by data.

Can we make a Majorana field theory starting from the zero mass Weyl theory, then adding a mass term as an interaction?

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The answer to this question is: yes we can. One can proceed similarly to the case of the Dirac massive field theory. In both cases one can start from the zero mass Weyl theory and then add a mass term as an interacting term of massless particles with a constant (external) field. In both cases the interaction gives rise to a field theory for a free massive fermion field. We shall present the procedure for the creation of a mass term in the case of the Dirac and the Majorana field and we shall look for a massive field as a superposition of massless fields. The Majorana particle is an unusual particle, since it is his own antiparticle.

1 The zero mass Weyl field theory

Let as start with massless particles, described by the Weyl massless fields. We pay attention on momentum ($p^a = (p^0, \vec{p})$) and spin of fields. Charges of fields will not be pointed out. It means that in our considerations we shall not distinguish between neutrinos, electrons and quarks. The Weyl equation for massless fields

$$2\vec{S}.\vec{p} = \Gamma p^0, \tag{4.1}$$

determines four states. Here $S^i = \frac{1}{2}\varepsilon_{ijk}S^{jk}$, $S^{ij} = \frac{i}{4}\,[\gamma^i, \gamma^j]$, $(i \in \{1,2,3\})$, which are the generators of the Lorentz transformations, determine the spin of states and γ^a, $a \in \{0,1,2,3\}$ are the Dirac operators. Operator $\Gamma = \frac{-i}{3!}\varepsilon_{abcd}S^{ab}S_{cd}$ is one of the two Casimir operators of the Lorentz group $SO(1,3)$ acting in the internal space of spins only and defines the handedness of states. Two eigenstates of Eq.(1.1) have left handedness ($<\Gamma>= r,\ r = -1$), the other two have right handedness[1] ($r = 1$). The left handed solutions have either left helicity

$$h = \frac{2\vec{S}.\vec{p}}{|\vec{p^0}|}, \tag{4.2}$$

($h = -1$) and positive energy ($p^0 = |p^0|$) or right helicity ($h = 1$) and negative energy ($p^0 = -|p^0|$). The right handed solutions have either right helicity ($h = 1$) and positive energy or left helicity ($h = -1$) and negative energy. We shall denote the positive energy solutions by a symbol u and the negative energy solutions by a symbol v. To determine the positive energy solution completely is enough to tell the momentum $\vec{p}$, ($p^0 = |\vec{p}|$) and either handedness or helicity: $u_{\vec{p},L} \equiv u_{\vec{p},h=-1}$, $u_{\vec{p},R} \equiv u_{\vec{p},h=1}$. Equivalently it follows : $v_{\vec{p},L} \equiv v_{\vec{p},h=1}$, $v_{\vec{p},R} \equiv v_{\vec{p},h=-1}$. We shall point out either helicity (h) or handedness (r), depending on what will be more convenient.

After quantizing the field the creation operators are defined, creating the negative energy particles: $d^{(0)+}_{\vec{p},L} \equiv d^{(0)+}_{\vec{p},h=1}$, $d^{(0)+}_{\vec{p},R} \equiv d^{(0)+}_{\vec{p},h=-1}$, and the positive energy particles: $b^{(0)+}_{\vec{p},L} \equiv b^{(0)+}_{\vec{p},h=-1}$, $b^{(0)+}_{\vec{p},R} \equiv b^{(0)+}_{\vec{p},h=1}$. The field can then according to ref.[1] be written as:

$$\psi(x) = \sum_{r=\pm1}\sum_{\vec{p},p^{02}=\vec{p}^2} \frac{1}{\sqrt{(2\pi)^3}}(b^{(0)}{}_{\vec{p},r}u_{\vec{p},r}e^{-ipx} + d^{(0)}{}_{\vec{p},r}v_{\vec{p},r}e^{ipx}). \tag{4.3}$$

To simplify the discussions we discretize the momentum and replace the integral with the sum. Then the energy operator $H^{(0)} = \int d^3\vec{x}\psi^+ p_0 \psi$ can be written as

$$H^{(0)} = \sum_{r=\pm1}\sum_{\vec{p},p^{02}=\vec{p}^2} |p^0|\ (b^{(0)+}_{\vec{p},r}b^{(0)}_{\vec{p},r} - d^{(0)+}{}_{\vec{p},r}d^{(0)}_{\vec{p},r}). \tag{4.4}$$

If the "totally empty" vacuum state is denoted by $|0>$, then the vacuum state occupied by massless particles up to $\vec{p} = 0$ is (due to discrete values of momenta) equal to

$$|\phi_{(0)}> = \prod_{\vec{p},r} d^{(0)+}_{\vec{p},r}|0> . \tag{4.5}$$

[1] We shall make use of the symbol Γ for the operator and r (ročnost in slovenian language means handedness) for the corresponding eigenvalue. The symbol h will be used for both, for the helicity operator and for his eigenvalue.

The energy of such a vacuum is accordingly $<\phi_0|H^{(0)}|\phi_0>=\sum_{\vec{p},r} E^{(0)}_{\vec{p},r}$, with $E^{(0)}_{\vec{p},r}=-|\vec{p}|$, which is of course infinite. Accordingly the particle state of momentum $\vec{p}, \acute{p}^0=|\vec{p}|$, and handedness r, with the energy, which is for $\acute{p}^0$ larger than the energy of the vacuum, can be written as $b^{(0)+}_{\vec{p}}|\phi_0>$.

2 Charge conjugation

The symmetry operation of charge conjugation is associated with the interchange of particles and antiparticles. Introducing the charge conjugation operator C, with the properties $C^2=C, C^+=C, C\gamma^{a*}C^{-1}=-\gamma^a$, where $(^+)$ stays for hermitian conjugation and $(^*)$ for complex conjugation, one finds the charge conjugated field $\psi(x)^C$ to the field $\psi(x)$ as $\psi(x)^C=C\psi(x)$. One accordingly finds for the charge conjugating operator $\mathcal{C}$, which affects creation and anihilation operators

$$\begin{array}{ll} \mathcal{C}\ b^{(0)+}_{\vec{p},h=-1}\ \mathcal{C}^{-1}=-d^{(0)}_{-\vec{p},h=1}, & \mathcal{C}\ b^{(0)+}_{\vec{p},h=1}\ \mathcal{C}^{-1}=d^{(0)}_{-\vec{p},h=-1}, \\ \mathcal{C}\ d^{(0)+}_{\vec{p},h=1}\ \mathcal{C}^{-1}=-b^{(0)}_{-\vec{p},h=-1}, & \mathcal{C}\ d^{(0)+}_{\vec{p},h=-1}\ \mathcal{C}^{-1}=b^{(0)}_{-\vec{p},h=1}. \end{array} \tag{4.6}$$

According to Sect. 2. the left handed column concerns the charge conjugation of left handed particles while the right handed column concerns the charge conjugation of right handed particles. One easily finds that the hamiltonian $H^{(0)}$ is invariant under the charge conjugation operation. The charge conjugation operation on the vacuum state $|\phi_{(0)}>=\prod_{\vec{p},r=\pm1} d^{(0)+}_{\vec{p},r}|0>$ should let it be invariant, since we want it as the physical vacuum to be charge conjugation invariant. To achieve that we are, however, then forced to let the totally empty vacuum $|0>$ transform under charge conjugation as

$$\mathcal{C}|0>=\prod_{\vec{p},r}(b^{\dagger}_{\vec{p},r}d^{\dagger}_{\vec{p},r}|0> \tag{4.7}$$

The charge conjugated operator $b^{(0)+}_{\vec{p},L}$ (which generates on a vacuum $|\phi_{(0)}>$ a one particle positive energy state of left helicity) annihilates in the vacuum state $|\phi_{(0)}>$ a negative energy particle state of opposite momentum and helicity and therefore generates a hole, which manifests as an antiparticle. Handedness stays unchanged.

3 The massive Dirac field theory

We shall first treat the case of the massive Dirac field, for which the procedure is simpler than in the massive Majorana case and from which we can learn the procedure. The mass term $\int d^3\vec{x} m_D\bar{\psi}\psi=\int d^3\vec{x}\ m_D\ (\bar{\psi_L}\psi_R+\bar{\psi_R}\psi_L)$ can be written, if using the expression for ψ from Eq.(4.3), as follows

$$H^{(1D)}=m_D\int d^3\vec{x}\ \bar{\psi}\psi=\sum_{h=\pm1}\sum_{\vec{p}} H^{(1D)}_{\vec{p},h},\quad H^{(1D)}_{\vec{p},h}=m_D\ (\ b^{(0)+}_{\vec{p},h}d^{(0)}_{\vec{p},h}+d^{(0)+}_{\vec{p},h}b^{(0)}_{\vec{p},h}\ ). \tag{4.8}$$

If we define

$$N_{\vec{p},h}=h^+_{\vec{p},h}+h^-_{\vec{p},h},$$

$$h^+_{\vec{p},h}=b^{(0)+}_{\vec{p},h}b^{(0)}_{\vec{p},h},\quad h^-_{\vec{p},h}=d^{(0)+}_{\vec{p},h}d^{(0)}_{\vec{p},h}, \tag{4.9}$$

one easily finds that $[N_{\vec{p},h},H^{(0)}_{\vec{\acute{p}},\acute{h}}]=0=[N_{\vec{p},h},H^{(1D)}_{\vec{\acute{p}},\acute{h}}]$. We see that the interaction term $H^{(1D)}_{\vec{p},h}$ does not mix massless states of different helicity. The appropriate basic states, which are eigenstates of the operator for number of particles of definite helicity $N_{\vec{p},h}$ are accordingly defined either with $b^{(0)+}_{\vec{p},h=1}$ and $d^{(0)+}_{\vec{p},h=1}$ with $h=1$ (but of right and left handedness, respectively) or with $b^{(0)+}_{\vec{p},h=-1}$ and $d^{(0)+}_{\vec{p},h=-1}$ with $h=-1$ (but of left and right handedness, respectively). The first two basic states have $<N_{\vec{p},h=1}>=1$ and $<N_{\vec{p},h=-1}>=0$, while the second two basic states have $<N_{\vec{p},h=1}>=0$ and $<N_{\vec{p},h=-1}>=1$. Diagonalizing $H^{(D)}_{\vec{p},h}=H^{(0)}_{\vec{p},h}+H^{(1D)}_{\vec{p},h}$ within the two basic states of definite helicity (but not handedness), one finds that

$$\begin{array}{ll} b^+_{\vec{p},h}=\alpha_{\vec{p}}\ b^{(0)+}_{\vec{p},h}+\beta_{\vec{p}}\ d^{(0)+}_{\vec{p},h}, & p^0=\ \ |p^0|, \\ d^+_{\vec{p},h}=\alpha_{\vec{p}}\ d^{(0)+}_{\vec{p},h}-\beta_{\vec{p}}\ b^{(0)+}_{\vec{p},h}, & p^0=-|p^0|, \\ \alpha_{\vec{p}}=\sqrt{\frac{1}{2}(1+\frac{|\vec{p}|}{|p^0|})}, & \beta_{\vec{p}}=\sqrt{\frac{1}{2}(1-\frac{|\vec{p}|}{|p^0|})}, \\ |p^0|=\sqrt{\vec{p}^2+m_D{}^2}. & \end{array} \tag{4.10}$$

The operator $b^{+}_{\vec{p},h}$ creates a massive positive energy one particle state ($p^0 = |p^0|$) and $d^{+}_{\vec{p},h}$ creates a massive negative energy one particle state ($p^0 = -|p^0|$), both states have momentum $\vec{p}$ and helicity h. Both are eigenstates of the hamiltonian for a massive Dirac field

$$H^{(D)} = \sum_{\vec{p},h} H^{(D)}_{\vec{p},h} = \sum_{\vec{p},h} (H^{(0)}_{\vec{p},h} + H^{(1D)}_{\vec{p},h}) = \sum_{\vec{p},h} |p^0|(b^{+}_{\vec{p},h} b_{\vec{p},h} - d^{+}_{\vec{p},h} d_{\vec{p},h}) \tag{4.11}$$

of momentum $\vec{p}$ and helicity h. The Dirac sea of massive particles is now

$$|\phi_{(D)}> = \prod_{\vec{p},h=\pm 1} d^{+}_{\vec{p},h}|0> = \pi_{\alpha}\; e^{-\sum_{\vec{p}} \frac{\beta_{\vec{p}}}{\alpha_{\vec{p}}} b^{(0)+}_{\vec{p},h} d^{(0)}_{\vec{p},h}}\; |\phi_{(0)}>, \; \pi_{\alpha} = \prod_{\vec{p}} \alpha_{\vec{p}p}. \tag{4.12}$$

All states up to $p^0 = -m_D$ are occupied and due to Eq.(4.7) it follows that $\mathcal{C}|\phi_{(D)}> = |\phi_{(D)}>$. The interaction term $H^{(1D)}$ causes the superposition of positive and negative energy massless states (Eq.(4.10)). One sees that the vacuum state of massive particles can be understood as a coherent state of particle and antiparticle pairs on the massless vacuum state. The energy of the vacuum state of massive Dirac particles $<\phi_{(D)}|H^{(D)}|\phi_{(D)}> = \sum_{\vec{p},h} E^{(D)}_{\vec{p},h}$, with $E^{(D)}_{\vec{p},h} = -\sqrt{\vec{p}^2 + m_D{}^2}$, which is infinite.

According to Eq.(4.10), the creation and annihilation operators for massive fields go in the limit when $m_D \longrightarrow 0$ to the creation and annihilation operators for the massless case.

A one particle state of energy $|p^0| = \sqrt{\vec{p}^2 + m_D^2}$ can be written as $b^{+}_{\vec{p},h}\,|\phi_{(D)}>$, with $b^{+}_{\vec{p},h}$ defined in Eq.(4.10). Also this state becomes in the limit $m_D = 0$ a massless Weyl one particle state of positive energy $|\vec{p}|$ above the Sea of massless particles.

One easily finds that $H^{(1D)}$ is invariant under charge conjugation and so is therefore also $H^{(D)}$. Taking into account Eqs.(4.6) it follows

$$\begin{array}{ll} \mathcal{C}\, b^{+}_{\vec{p},h=-1}\, \mathcal{C}^{-1} = -d_{-\vec{p},h=1}, & \mathcal{C}\; b^{+}_{\vec{p},h=1}\; \mathcal{C}^{-1} = d_{-\vec{p},h=-1}, \\ \mathcal{C}\; d^{+}_{\vec{p},h=1}\; \mathcal{C}^{-1} = -b_{-\vec{p},h=-1}, & \mathcal{C}\; d^{+}_{\vec{p},h=-1}\, \mathcal{C}^{-1} = b_{-\vec{p},h=1}. \end{array} \tag{4.13}$$

In the limit $m_D \longrightarrow 0$ Eqs. (4.13) coincide with Eqs. (4.6). The charge conjugation transforms the particle of a momentum $\vec{p}$ and helicity h into the hole in the Dirac sea of the momentum $-\vec{p}$ and helicity $-h$.

4 The Majorana massive field theory

The Majorana mass term with only left handed fields $m_{ML} \int d^3\vec{x}\; (\bar{\psi}_L + \bar{\psi}_L{}^C)(\psi_L + \psi_L^C)$ can be written, if using the expression for ψ from Eq.(1.3), with the summation going over the left handed fields only and if taking into account the definition of charge conjugation from Sect. 1., as follows

$$H^{(1M)}_L = m_{ML} \int d^3\vec{x}\; (\bar{\psi}_L + \bar{\psi}_L{}^C)(\psi_L + \psi_L{}^C) = \sum_{(\vec{p})^+} H^{(1M)}_{\vec{p},L},$$

$$H^{(1M)}_{\vec{p},L} = m_{ML}\; (b^{(0)+}_{\vec{p},h=-1} b^{(0)+}_{-\vec{p},h=-1} + b^{(0)}_{-\vec{p},h=-1} b^{(0)}_{\vec{p},h=-1} + d^{(0)+}_{\vec{p},h=1} d^{(0)+}_{-\vec{p},h=1} + d^{(0)}_{-\vec{p},h=1} d^{(0)}_{\vec{p},h=1}). \tag{4.14}$$

The symbol $\sum_{(\vec{p})^+}$ means that the sum runs over $\vec{p}$ on such a way that $\vec{p}$ and $-\vec{p}$ is counted only once. Comparing the Majorana interaction term $H^{(1M)}{}_L$ of Eq.4.14 with the Dirac interaction term of Eq.4.8, one sees that in both cases momentum $\vec{p}$ is conserved as it should be. In Eq. 4.14 the two creation operators appear with opposite momentum, while in Eq.4.8 the creation and annihilation operator appear with the same momentum. Because of that we could pay attention in the Dirac case to a momentum $\vec{p}$, without connecting $\vec{p}$ with $-\vec{p}$, while in the Majorana case we have to treat $\vec{p}$ and $-\vec{p}$ at the same time.

The Majorana mass term of only right handed fields follows from the mass term of only left handed fields of Eq.4.14 if we exchange $h = -1$ with $h = 1$ and $h = 1$ with $h = -1$. We shall treat here the left handed fields only. The corresponding expressions for the massive Majorana right handed fields can be obtained from the left handed ones by the above mentioned exchange of helicities of fields.

It is easy to check that the charge conjugation operator $\mathcal{C}$ from Eq. 4.6 leaves the interaction term of Eq.4.14 unchanged. Accordingly also the hamiltonian

$$H^{(M)}_{\vec{p},L} = H^{(0)}_{\vec{p},L} + H^{(1M)}_{\vec{p},L} \tag{4.15}$$

is invariant under charge conjugation: $[H^{(M)}_{\vec{p},L}, \mathcal{C}] = 0$.

As in the Dirac massive case, it is meaningful to use the operators $h^{+}_{\vec{p},h=-1} = b^{(0)+}_{\vec{p},h=-1} b^{(0)}_{\vec{p},h=-1}$ and $h^{-}_{\vec{p},h=1} = d^{(0)+}_{\vec{p},h=1} d^{(0)}_{\vec{p},h=1}$, to choose the appropriate basis within which we shall diagonalize the hamiltonian of Eq.4.14. One can check that the operators

$$h^{+}_{\vec{p}} = h^{+}_{\vec{p},h=-1} - h^{+}_{-\vec{p},h=-1}, \quad h^{-}_{\vec{p}} = h^{-}_{\vec{p},h=1} - h^{-}_{-\vec{p},h=1}, \tag{4.16}$$

which count the momentum of states, commute with the hamiltonian 4.15

$$[h^{\pm}_{\vec{p}}, H^{(M)}_{\vec{p},L}] = 0. \tag{4.17}$$

Since basic states, appropriate to describe the vacuum state, should have momentum equal zero to guarantee the zero momentum of the vacuum, one looks for the basic states with $< h^{\pm}_{\vec{p}} >= 0$. One finds four such states

$$\begin{array}{rc} |1>= & b^{(0)+}_{\vec{p},h=-1} b^{(0)+}_{-\vec{p},h=-1}|0>, \\ |2>= & \frac{1}{\sqrt{2}}(1 + b^{(0)+}_{\vec{p},h=-1} b^{(0)+}_{-\vec{p},h=-1} d^{(0)+}_{\vec{p},h=1} d^{(0)+}_{-\vec{p},h=1})|0>, \\ |3>= & d^{(0)+}_{\vec{p},h=1} d^{(0)+}_{-\vec{p},h=1}|0> \\ \hline |4>= & \frac{1}{\sqrt{2}}(1 - b^{(0)+}_{\vec{p},h=-1} b^{(0)+}_{-\vec{p},h=-1} d^{(0)+}_{\vec{p},h=1} d^{(0)+}_{-\vec{p},h=1})|0>. \end{array} \tag{4.18}$$

One finds that the state $|4>$ is the eigenstate of the hamiltonian of Eq.4.15 with the eigenvalue zero. The hamiltonian applied on first three basic states defines a matrix

$$\left(\begin{array}{ccc} 2|\vec{p}| & \sqrt{2}m & 0 \\ \sqrt{2}m & 0 & \sqrt{2}m \\ 0 & \sqrt{2}m & -2|\vec{p}| \end{array} \right). \tag{4.19}$$

Diagonalizing this matrix one finds three vectors and three eigenvalues. The only candidate for the vacuum state is the state $\beta_{\vec{p}}{}^2|1> +(-)\alpha_{\vec{p}}\beta_{\vec{p}}|2> +\alpha_{\vec{p}}{}^2|3>$, with $\alpha_{\vec{p}}$ and $\beta_{\vec{p}}$ defined in Eq.4.10, with the eigenvalue $-2\sqrt{|\vec{p}|^2 + m_{ML}{}^2}$ which corresponds to the vacuum state of the $-2\sqrt{|\vec{p}|^2 + m_D{}^2}$ energy in the Dirac massive case $d^{+}_{\vec{p},h=1} d^{+}_{-\vec{p},h=1}|0>$. The Majorana vacuum state is accordingly

$$|\phi_{(ML)}>= \prod_{(\vec{p})^+} |\phi_{M\vec{p},L}>, \; |\phi_{M\vec{p},L}>= (\beta_{\vec{p}}{}^2|1> +(-)\alpha_{\vec{p}}\beta_{\vec{p}}|2> +\alpha_{\vec{p}}{}^2|3>), \tag{4.20}$$

with $\alpha_{\vec{p}}$ and $\beta_{\vec{p}}$ defined in Eq.4.10. Compared to the Dirac particle case it should be noted that we for Majorana had to combine both $vecp$ and $-vecp$ under the diagonalization and construction of ground state 4.20. The energy of the vacuum of Majorana left handed particles is $< \phi_{(ML)}|H^{(M)}|\phi_{(ML)} >= \sum_{(\vec{p})^+,L} E^{(ML)}_{\vec{p},L}$, with $E^{(ML)}_{\vec{p},L} = -2\sqrt{\vec{p}^2 + m_{ML}{}^2}$, which is the energy of two majorana particles of momentum $p^a = (-|p^0|, \vec{p})$ and $p^a = (-|p^0|, -\vec{p})$, respectively. The energy of the Majorana sea is again infinite. In the limit $m_{ML} \longrightarrow 0$ the Majorana sea becomes a sea of massless Weyl particles of only left handedness.

Concerning charge conjugation we see that with the somewhat complicated transformation of the "totally empty vacuum" (Eq.4.7) the Majorana physical vacuum $|\phi_{ML}>$ is charge conjugation invariant

$$\mathcal{C}|\phi_{ML}>= |\phi_{ml}>. \tag{4.21}$$

The one particle Majorana states can be constructed as superpositions of states with $< h^{(+)}{}_{\vec{p}} + h^{(-)}{}_{\vec{p}} >= \pm 1$. One finds four times two states which fulfil this condition

$$
\begin{array}{lc}
|5>= & b^{(0)+}_{\vec{p},h=-1}|0>,\\
|6>= & b^{(0)+}_{\vec{p},h=-1}d^{(0)+}_{\vec{p},h=1}d^{(0)+}_{-\vec{p},h=1}|0>,\\
\hline
|7>= & d^{(0)+}_{\vec{p},h=1}|0>,\\
|8>= & d^{(0)+}_{\vec{p},h=1}b^{(0)+}_{\vec{p},h=-1}b^{(0)+}_{-\vec{p},h=-1}|0>,\\
\hline\hline
|9>= & b^{(0)+}_{-\vec{p},h=-1}|0>,\\
|10>= & b^{(0)+}_{-\vec{p},h=-1}d^{(0)+}_{\vec{p},h=1}d^{(0)+}_{-\vec{p},h=1}|0>,\\
\hline
|5>= & d^{(0)+}_{-\vec{p},h=1}|0>,\\
|6>= & d^{(0)+}_{-\vec{p},h=-1}b^{(0)+}_{\vec{p},h=-1}b^{(0)+}_{-\vec{p},h=-1}|0>,
\end{array}
\tag{4.22}
$$

The first four states have a momentum $\vec{p}$ and the last four states a momentum $-\vec{p}$. The hamiltonian $H^{(M)}\vec{p},L$ defines on these states the block diagonal four two by two matrices. The candidates for the states describing a one particle state of momentum $\vec{p}$ on a vacuum states $|0>$ are states with energy which is for $p^0=\sqrt{\vec{p}^2+m^2_{ML}}$ higher than the vacuum state. One finds the corresponding operators

$$
\begin{array}{lc}
b^{+}_{\vec{p},h=-1}= & -\beta_{\vec{p}}\, b^{(0)+}_{\vec{p},h=-1}+\alpha_{\vec{p}}\, b^{(0)+}_{\vec{p},h=-1}d^{(0)+}_{\vec{p},h=1}d^{(0)+}_{-\vec{p},h=1},\\
d^{+}_{\vec{p},h=1}= & -\alpha_{\vec{p}}\, d^{(0)+}_{\vec{p},h=1}+\beta_{\vec{p}}\, d^{(0)+}_{\vec{p},h=1}b^{(0)+}_{\vec{p},h=-1}b^{(0)+}_{-\vec{p},h=-1},\\
\hline
b^{+}_{-\vec{p},h=-1}= & -\beta_{\vec{p}}\, b^{(0)+}_{-\vec{p},h=-1}+\alpha_{\vec{p}}\, b^{(0)+}_{-\vec{p},h=-1}d^{(0)+}_{\vec{p},h=1}d^{(0)+}_{-\vec{p},h=1},\\
d^{+}_{-\vec{p},h=1}= & -\alpha_{\vec{p}}\, d^{(0)+}_{-\vec{p},h=1}+\beta_{\vec{p}}\, d^{(0)+}_{-\vec{p},h=1}b^{(0)+}_{\vec{p},h=-1}b^{(0)+}_{-\vec{p},h=-1},
\end{array}
\tag{4.23}
$$

which when applied on a true vacuum $|0>$ generates the one particle states of momentum $\vec{p}$ (the first two operators) and $-\vec{p}$ (the second two operators), respectively.

We would prefer to know, as in the Dirac massive case, the one particle operators which when being applied on a Majorana vacuum state $|\phi_{(ML)}>$ generates a one particle Majorana state with chosen momentum $\vec{p}$ and which commute with the charge conjugate operator $\mathcal{C}$ defined in Eq.4.6. Requiring $B^{+}{}_{\vec{p},h=-1}|\phi_{\vec{p},L}>=b^{+}_{\vec{p},h=-1}|0>$ one finds $B^{+}{}_{\vec{p},h=-1}=\alpha_{\vec{p}}\, b^{(0)+}_{\vec{p},h=-1}+\beta_{\vec{p}}\, b^{(0)}_{-\vec{p},h=-1}$. Accordingly it follows from the requirement $D^{+}{}_{\vec{p},h=1}|\phi_{\vec{p},L}>=d^{+}_{\vec{p},h=1}|0>$ that $D^{+}{}_{\vec{p},h=1}=\beta_{\vec{p}}\, d^{(0)+}_{\vec{p},h=1}+\alpha_{\vec{p}}\, d^{(0)}_{-\vec{p},h=1}$. Taking into account that $\mathcal{C}\ B^{+}{}_{\vec{p},h=-1}\mathcal{C}^{-1}=-D^{+}_{-\vec{p},h=1}$ we may conclude that the two operators

$$
\mathcal{B}^{+}{}_{\pm\vec{p},h=-1}=\alpha_{\vec{p}}\,(b^{(0)+}_{\pm\vec{p},h=-1}-d^{(0)}_{\mp\vec{p},h=1})-\beta_{\vec{p}}\,(d^{(0)+}_{\pm\vec{p},h=1}-b^{(0)}{}_{\mp\vec{p},h=-1}).
\tag{4.24}
$$

$\mathcal{B}^{+}{}_{\pm\vec{p},h=-1}$ operating on the Majorana vacuum state $|\phi_{ML}>$ generates the one particle Majorana state of momentum $\pm\vec{p}$. It can easily be checked that Majorana particle is his own antiparticle $\mathcal{C}\ \mathcal{B}^{+}{}_{\pm\vec{p},h=-1}\ \mathcal{C}^{-1}=\mathcal{B}^{+}{}_{\pm\vec{p},h=-1}$.

In the limit when $m_{ML}\longrightarrow 0$, the operator $\mathcal{B}^{+}{}_{\pm\vec{p},h=-1}$ operating on a vacuum $|\phi_{(ML)}>$, which goes to the vacuum state of the massless case of only left handed paricles gives a state of a Majorana massless particle: $(b^{(0)+}{}_{\pm\vec{p},h=-1}-d^{(0)}_{\mp\vec{p},h=1})\ d^{(0)+}{}_{\vec{p},h=1}d^{(0)+}{}_{-\vec{p},h=1}\prod_{\vec{p},\acute{p}\neq p}\ d^{(0)+}{}_{\vec{\acute{p}}}|0>$.

One can accordingly find the operators for right handed Majorana particles.

We have learned that it is indeed possible to define the Majorana sea in the way the Dirac sea is defined. This put a new light on the Majorana particle. It stays to study whether or not this presentation can be used to better understand the properties of the Majorana particles.

References

[1] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields*, McGraw-Hill Book Company, New York 1965

Can one connect the Dirac-Kähler representation of Dirac spinors and spinor representations in Grassmann space, proposed by Mankoč?

Holger and Norma

The contribution to this question which follows runs out of a complaint like this by Holger and others:
It is very suspicious that you, Norma, get Dirac spinors out of a start with only θ^a variables, which are clearly vectors under Lorentz transformations: wave functions depending on the θ^a-variables have a priori no way of being spinors.
Really this is much like the Dirac-Kähler construction.

1 Introduction:

In Norma's contribution to these proceedings the fermions come out of the d dimensional Grassmannian set of coordinates in the sense that the spin degrees of freedom of the fermions appear as represented by wave functions defined on this space parametrized by Grasmannian variables called there θ^a. For obtaining the **spinor** degrees of freedom the ordinary spatio-temporal coordinates x^a also assumed in Norma's model are not important and we here totally ignore those. The variables θ^a were assumed to transform as a vector under Lorentz transformations. One would therefore - a priori - observe that all wave functions depending on these θ^a's could only transform as having integer spin. Nevertheless Norma claims to get half integer spin degrees of freedom out of her model from these variables!
This mysterious result comes about after introducing for the Grassmann odd variables the creation and anihilation like operators

$$\tilde{a}^a := i(p^{\theta a} - i\theta^a), \quad \tilde{\tilde{a}}^a := -(p^{\theta a} + i\theta^a), \tag{5.1}$$

where

$$p^{\theta a} := -i\overrightarrow{\partial^{\theta}}_a. \tag{5.2}$$

A crucial - and by Holger (but not Norma) considered suspicious - step consists in hoping for that a "constraint" (or whatever) could make the $\tilde{\tilde{a}}^a$ vanish so as to justify putting

$$\tilde{\tilde{a}}^a \to 0 \tag{5.3}$$

whenever it occurs in the Hamiltonian (See Norma's contribution Eq.(4.8)). The excuse for this replacement is that it is easily found that

$$\{\tilde{a}^a, \tilde{\tilde{a}}^a\} = 0 \tag{5.4}$$

and also that therefore

$$[\tilde{S^{ab}}, \tilde{\tilde{S}}cd] = 0 \tag{5.5}$$

where the Lorentz transformation generator parts $\tilde{S}^{ab}$ and $\tilde{\tilde{S}}cd$ are defined by

$$\tilde{S}^{ab} := \frac{i}{4}[\tilde{a}^a, \tilde{a}^b] \;\; ; \;\; \tilde{\tilde{S}}cd := \frac{i}{4}[\tilde{\tilde{a}}^c, \tilde{\tilde{a}}^d] \tag{5.6}$$

and together make up the total Lorentz generator

$$S^{ab} = \tilde{S}^{ab} + \tilde{\tilde{S}}ab. \tag{5.7}$$

In this way it could be consistent if all the $\tilde{\tilde{a}}^a$ were set to zero because they commute with the rest of the variables - the $\tilde{a}^a$. But the best reason for seeking to put the $\tilde{\tilde{a}}^a$ to zero is that with a reasonable choice of the kinetic term in the Lagrangian $-i\dot{\theta}^a\theta^a$ we get as the expression for the conjugate variable to θ^a

$$p^{\theta a} = i\theta^a \tag{5.8}$$

which implies that

$$\tilde{\tilde{a}}^a = 0. \tag{5.9}$$

(See Norma's contribution Eq.(4.2))
Now, however, the way that Norma chooses to quantize the system, that is a particle moving in (ordinary and) Grassmanian coordinate space, is to let the wave function be allowed to be any function of the d Grassmann variables θ^a, so that any such function represents a state of the system. But in this quantization the $\tilde{\tilde{a}}^a$ turn out not to be zero. In other words that quantization turned out not to obey the equation expected from expression for the canonical coordinate $p^{\theta a}$ as derived from the Lagrangian.
If, however, in the operators such as the Hamiltonian and the Lorentz transformation operators $\tilde{\tilde{a}}^a$ are just put to zero, the expressions obtained after having put the $\tilde{\tilde{a}}^a$ to zero - i.e. we especially only use $\tilde{S}^{ab}$ as the Lorentz generator - one has in principle a new Lorentz transformation instead of the a priori one in the wave function on Grassmann-space quantization used. In that case one could a priori expect that the argument for only having integer spin could break down. Indeed the calculations confirm this to happen.
We should now attempt to get an understanding of what goes on here by using a basis inspired from the Dirac-Kähler construction, which is a way often used on lattices to implement fermions on the lattice. The Dirac-Kähler constructions starts from a field theory with a series of fields which are 0-form, 1-form, 2-form, ...,d-form. They can be thought of as being expanded on a basis of all the wedge product combinations of the basis dx^1, dx^2, ...,dx^d for the one-forms, including wedge products from zero factors to d factors. In the Dirac-Kähler construction one succeeds in constructing out of these"all types of forms" for $d = 2n$, with n an integer, $2^{d/2}$ Dirac spinor fields. This construction should without cheat be impossible in much the same way as Norma's ought to be.
It is the major point of the below calculation to use the ideas of the Dirac-Kähler construction to make such a basis choice for Norma's wavefunctions that the connection between the two seemingly impossible achievements - Dirac-Kählers and Norma's - are brought to more light.

2 Dirac-Kähler approach in our own way

2.1 The basis with the spinor indices

With the Dirac-Kähler construction in mind we propose to expand the set of wave functions that was assumed to be the set of all functions of the d Grassmann variables θ^a on the following system of basis wave functions, that are marked by two spinor indices and for the case of an odd d in addition by an index Γ which can take two values $+1$ and -1 and which reminds of the handedness.
For the even d case one has

$$\psi_{\alpha\beta}(\{\theta^a\}) := \sum_{i=0}^{d} (\gamma_{a_1}\gamma_{a_2}\cdots\gamma_{a_i})_{\alpha\beta}\theta^{a_1}\theta^{a_2}\cdots\theta^{a_i}, \tag{5.10}$$

while for the odd d case :

$$\psi_{\alpha\beta\Gamma}(\{\theta^a\}) := \sum_{i=0}^{d} (\gamma_{(\Gamma)a_1}\gamma_{(\Gamma)a_2}\cdots\gamma_{(\Gamma)a_i})_{\alpha\beta}\theta^{a_1}\theta^{a_2}\cdots\theta^{a_i}, \tag{5.11}$$

with the convention $a_1 < a_2 < a_3 < ... < a_i$. Here the sums run over the number i of factors in the products of θ^a coordinates, a number which is the same as the number of gamma-matrix factors and it should be remarked that we include the possibility $i = 0$ which means no factors and is taken to mean that the product of the θ^a-factors is unity and the product of zero gamma matrices is the unit matrix (as is natural). The indices $\alpha\beta$ are the spinor indices and taking the product of gamma matrices as matrices the symbol $(...)_{\alpha\beta}$ means taking the αth row and βth column element in the matrix. There is an understood Einstein convention summation over the contracted indices a_1,a_2, ...,a_i, which are vector indices. The gamma-matrices are in the even case $2^{d/2}$ by $2^{d/2}$ matrices and in the odd case $2^{(d-1)/2}$ by $2^{(d-1)/2}$ matrices, but in the latter case we can choose the sign on say the last one of these gamma-matrices depending on the last wavefunction basis index Γ so as to arrange that

$$\gamma_1\gamma_2\cdot\cdots\cdot\gamma_d = \Gamma. \tag{5.12}$$

The γ^a matrices could be constructed as follows - a construction also showing that they really do exist - by an easy check of the Clifford algebra

$$\{\gamma_a, \gamma_b\} = 2\eta_{ab} \tag{5.13}$$

using a $2^{d/2}$ or $2^{(d-1)/2}$ dimensional spinor space conceived of as the cartesian product $d/2$ or $(d-1)/2$ spin one half two dimensional Hilbert spaces:

$$
\begin{array}{ccccccccccc}
\gamma_1 & := & i\sigma_2^1 & \times & \sigma_3^2 & \times & \sigma_3^3 & \times & \cdots & \times & \sigma_3^n \\
\gamma_2 & := & -i\sigma_1^1 & \times & \sigma_3^2 & \times & \sigma_3^3 & \times & \cdots & \times & \sigma_3^n \\
\gamma_3 & := & iI^1 & \times & \sigma_2^2 & \times & \sigma_3^3 & \times & \cdots & \times & \sigma_3^n \\
\gamma_4 & := & iI^1 & \times & (-)\sigma_1^2 & \times & \sigma_3^3 & \times & \cdots & \times & \sigma_3^n \\
\gamma_5 & := & iI^1 & \times & I^2 & \times & \sigma_2^3 & \times & \cdots & \times & \sigma_3^n \\
\vdots & & \vdots & \vdots & \vdots & \ddots & \vdots & & & & \\
\gamma_{2n-1} & := & iI^1 & \times & I^2 & \times & I^3 & \times & \cdots & \times & \sigma_2^n \\
\gamma_{2n} & := & iI^1 & \times & I^2 & \times & I^3 & \times & \cdots & \times & (-)\sigma_1^n
\end{array}
$$

for an even $d = 2n$, while for an odd $d = 2n + 1$ the term γ_{2n+1} has to be added as follows

$$
\gamma_{2n+1} \quad := \quad i\Gamma\sigma_3^1 \quad \times \quad \sigma_3^2 \quad \times \quad \sigma_3^3 \quad \times \quad \cdots \quad \times \quad \sigma_3^n,
$$

with $\Gamma = \prod_i^{2n+1} \gamma_i$.
The above metric is supposed to be Euclidean. For the Minkowski metric $\gamma_1 \to -i\gamma_1$ has to be taken, if the index 1 is recognized as the "time" index. We shall make use of the Minkowski metric, counting the γ^a from $0, 1, 2, 3, 5, ..d$, and assuming the metric $g^{ab} = diag(1, -1, -1, ..., -1)$.
It is our main point to show that the action by the operators $\tilde{a}^a$ and $\tilde{\tilde{a}}^a$ in the representation based on the basis $\psi_{\alpha\beta}(\{\theta^a\})$ with $\alpha, \beta = 1, 2, ..., \left\{ \begin{array}{c} 2^{(d-1)/2} \ \text{ for } \ d \ \text{ odd} \\ 2^{d/2} \ \text{ for } \ d \ \text{ even} \end{array} \right\}$ transforms the index α and β, respectively, of the basis $\psi_{\alpha\beta}(\{\theta^a\})$ as follows:

$$
\tilde{a}^a \psi_{\alpha\beta(\Gamma)}(\{\theta^a\}) \propto \gamma^a_{\alpha\gamma} \psi_{\gamma\beta(\Gamma)}(\{\theta^a\}), \tag{5.14}
$$

$$
\tilde{\tilde{a}}^a \psi_{\alpha\beta(\Gamma)}(\{\theta^a\}) \propto \psi_{\alpha\gamma(-\Gamma)}(\{\theta^a\}) \gamma^a_{\gamma\beta}, \tag{5.15}
$$

which demonstrates the similarities between the spinors of the Normas approach and the Dirac-Kähler approach: The operators $\tilde{a}^a$ transform the left index of the basis $\psi_{\alpha\beta(\Gamma)}(\{\theta^a\})$, while keeping the right index fixed and the operators $\tilde{\tilde{a}}^a$ transform the right index of the basis $\psi_{\alpha\beta(\Gamma)}(\{\theta^a\})$ and keep the left index fixed. Under the action of either $\tilde{a}^a$ or $\tilde{\tilde{a}}^a$ the basic functions transform as spinors. The index in parentheses (Γ) is defined for only odd d. We can count that the number of spinors is 2^d either in the Norma's approach (the d dimensional Grassmann space has 2^d basic functions) or in the Dirac-Kähler approach (for $d = 2n$ the number of spinors is $2^{d/2} \cdot 2^{d/2}$, while for $d = 2n + 1$ the number of spinors is $2^{(d-1)/2} \cdot 2^{(d-1)/2} \cdot 2$, n is an integer).
We shall prove the above formulas for action of the $\tilde{a}^a$ and $\tilde{\tilde{a}}^a$ on basic functions after presenting special cases $d = 1, d = 2$.

2.2 d=1,2 special cases checking basis properties

We shall present the Dirac-Kähler basis for two cases $d = 1, 2, 3$ to see what it means. For comparison with the basis of the Mankoč approach see ref.[4].
i) The one dimensional ($d = 1$) space.
The Dirac-Kähler basis has two (2^1) vectors of the mixed Grassmann character.

$$
\gamma^0 = \mathrm{I}\,\Gamma, \quad \psi_{11,\Gamma=1} = 1 + \theta, \quad \psi_{11,\Gamma=-1} = 1 - \theta. \tag{5.16}
$$

The operator $\tilde{a}^0$ is in this basis a diagonal and $\tilde{\tilde{a}}^0$ a non-diagonal matrix. The superposition of the above basis leads to the new basis $(1, \theta)$ with well defined Grassmann character.
ii) The two dimensional ($d = 2$) space.
The Dirac-Kähler basis has four (2^2) vectors of either even or odd Grassmann character. According to the definition of γ - matrices we have

$$
\gamma^0 = \sigma^2, \ \ \gamma^1 = -i\sigma^1 \tag{5.17}
$$

and

$$
\psi_{\alpha\beta} = 1_{\alpha\beta} + (\gamma^0)_{\alpha\beta}\theta^0 + (\gamma^1)_{\alpha\beta}\theta^1 + (\gamma^0\gamma^1)_{\alpha\beta}\theta^0\theta^1. \tag{5.18}
$$

One finds accordingly

$$
\psi_{11} = 1 - \theta^0\theta^1, \ \ \psi_{12} = -i(\theta^0 + \theta^1), \ \ \psi_{21} = i(\theta^0 - \theta^1), \ \ \psi_{22} = 1 + \theta^0\theta^1. \tag{5.19}
$$

One easily checks that $\tilde{a}^a$, $\ a \in 1, 2$, transform the first index of $\psi_{\alpha,\beta}$, while $\tilde{\tilde{a}}^a$, $\ a \in 1, 2$, transform the second index of $\psi_{\alpha,\beta}$, both transforming a Grassmann odd function into a Grassmann even function or opposite.

2.3 Proof of our formula for action of $\tilde{a}^a$ and $\tilde{\tilde{a}}^a$

Let us first introduce the notation

$$\gamma^A := \gamma^a \gamma^b \cdots \gamma^c, \;\; \gamma^{\overline{A}} := \gamma^c \gamma^b \cdots \gamma^a, \tag{5.20}$$

with $a < b < \cdots < c \in A$. We recognize that

$$\text{Trace } (\gamma_A \gamma^{\overline{B}}) = \text{Trace(I) } \delta_A{}^B, \;\; \sum_A (\gamma_A)_{\alpha\beta} (\gamma^{\overline{A}})_{\gamma\delta} = \text{Trace(I) } \delta_{\alpha\gamma} \; \delta_{\beta\delta} \tag{5.21}$$

and

$$\sum_i (\gamma_{A_i})_{\alpha\beta} (\gamma^c \gamma^{\overline{A}_i})_{\gamma\delta} = \text{Trace(I) } (\gamma^c)_{\alpha\delta} \delta_{\beta\gamma}, \;\;\; \sum_i (\gamma_{A_i})_{\alpha\beta} (\gamma^{\overline{A}_i} (-1)^i \gamma^c)_{\gamma\delta} = \text{Trace(I) } (\gamma^c)_{\gamma\beta} \delta_{\delta\alpha}. \tag{5.22}$$

Using the first equation we find

$$\theta^A = \frac{1}{\text{Trace(I)}} \; (\gamma^{\overline{A}})_{\alpha\beta} \; \psi_{\beta\alpha(\Gamma)}(\{\theta^a\}). \tag{5.23}$$

The index (Γ) has the meaning for only odd d. That is why we put it in parenthesis. We may accordingly write

$$\psi_{\alpha\beta(\Gamma)}(\{\theta^a\}) := \sum_{i=0} \frac{1}{\text{Trace(I)}} \; (\gamma_{A_i})_{\alpha\beta} (\gamma^{\overline{A}_i})_{\gamma\delta} \psi_{\delta\gamma(\Gamma)}(\{\theta^a\}), \tag{5.24}$$

with $A_i \in a_1 < a_2, \cdots, < a_i$ in ascending order and with $\overline{A}_i$ in descending order.
Then we find, taking into account that $\tilde{a}^a|0>= \theta^a$, $\tilde{\tilde{a}}^a|0>= -i\theta^a$, where $|0>$ is a vacuum state and Eq.(4)

$$\tilde{a}^c \psi_{\alpha\beta(\Gamma)}(\{\theta^a\}) := \sum_i (\gamma_{A_i})_{\alpha\beta} \tilde{a}^c \theta^{A_i} = \sum_i (\gamma_{A_i})_{\alpha\beta} \tilde{a}^c \tilde{a}^{A_i} |0>= \sum_i \frac{1}{\text{Trace(I)}} \; (\gamma_{A_i})_{\alpha\beta} (\gamma^c \gamma^{\overline{A}_i})_{\gamma\delta} \psi_{\delta\gamma(\Gamma)}(\{\theta^a\}). \tag{5.25}$$

Using the above relations we further find

$$\tilde{a}^c \psi_{\alpha\beta(\Gamma)}(\{\theta^a\}) := (-1)^{\tilde{f}(d,c)} (\gamma^c)_{\alpha\gamma} \psi_{\gamma\beta(\Gamma)}(\{\theta^a\}), \tag{5.26}$$

where $(-1)^{\tilde{f}(d,c)}$ is ± 1, which depends on the operator $\tilde{a}^c$ and the dimension of the space.
We find in a similar way

$$\tilde{\tilde{a}}^c \psi_{\alpha\beta(\Gamma)}(\{\theta^a\}) := \sum_i (\gamma_{A_i})_{\alpha\beta} \tilde{\tilde{a}}^c \tilde{a}^{A_i} |0>= \sum_i (-1)^i (\gamma_{A_i})_{\alpha\beta} \tilde{a}^{A_i} \tilde{\tilde{a}}^c |0>= \sum_i (-1)^i (\gamma_{A_i})_{\alpha\beta} \tilde{a}^{A_i} \tilde{a}^c |0>, \tag{5.27}$$

which gives

$$\sum_i \frac{(-1)^i}{\text{Trace(I)}}; \; (\gamma_{A_i})_{\alpha\beta} (\gamma^{\overline{A}_i \gamma^c})_{\gamma\delta} \; \psi_{\delta\gamma(\Gamma)}(\{\theta^a\}), \tag{5.28}$$

and finally

$$\tilde{\tilde{a}}^c \psi_{\alpha\beta(\Gamma)}(\{\theta^a\}) := (-1)^{\tilde{\tilde{f}}(d,c)} \psi_{\alpha\gamma(-\Gamma)}(\{\theta^a\}) (\gamma^c)_{\gamma\beta}, \tag{5.29}$$

with the sign $(-1)^{\tilde{\tilde{f}}(d,c)}$ depending on the dimension of the space and the operator $\tilde{\tilde{a}}^c$.
We have therefore proven the two equations which determines the action of the operators $\tilde{a}^a$ and $\tilde{\tilde{a}}^a$ on the basic function $\psi_{\alpha\gamma(-\Gamma)}(\{\theta^a\})$.

3 Getting an even gamma matrix

According to the Eqs.([**?**, t,tt] it is obvious that the γ^a **matrices, entering into the Dirac-Kähler approach for spinors, have an odd Grassmann character** since both, $\tilde{a}^a$ and $\tilde{\tilde{a}}^a$, have an odd Grassmann character. They therefore transform a Grassmann odd basic function into a Grassmann even basic function changing fermions into bosons. It is clear that such γ^a matrices are not appropriate to enter into the equations of motion and Lagrangeans for spinors.
Can one find an appropriate definition of the γ^a matrices? Yes. Here is an suggestion for the way out!
If working with $\tilde{a}^a$ only, putting $\tilde{\tilde{a}}^a$ in the Hamiltonian, Lagrangean and all the operators equal to zero, it is meaningful to define the γ^a matrices of an even Grassmann character as follows

$$\tilde{\gamma}^a := i\tilde{a}^a \tilde{\tilde{a}}^0. \tag{5.30}$$

One can immediately check that

$$\{\tilde{\gamma}^a, \tilde{\gamma}^b\} = \{\tilde{a}^a, \tilde{a}^b\} = 2\eta^{ab}, \quad \tilde{S}^{ab} = \frac{i}{4}[\tilde{a}^a, \tilde{a}^b] = \frac{i}{4}[\tilde{\gamma}^a, \tilde{\gamma}^b]. \tag{5.31}$$

We have

$$\tilde{\gamma}^a \psi_{\alpha\beta(\Gamma)}(\{\theta^a\}) = \gamma^a_{\alpha\gamma}\psi_{\gamma\delta(-\Gamma)}(\{\theta^a\})\gamma^0_{\delta\beta}. \tag{5.32}$$

One can check that $\tilde{\gamma}^a$ have all the properties of the Dirac γ^a matrices.
(If working only with $\tilde{\tilde{a}}^a$ the γ^a matrices defined as $\tilde{\tilde{\gamma}}^a := i\tilde{\tilde{a}}^a\tilde{a}^0$ have again all the properties of the Dirac γ^a matrices.)

4 Conclusion and what we learn

We have shown that the answer to the question: "**Can one connect the Dirac-Kähler representation of Dirac spinors and spinor representations in Grassmann space, proposed by Mankoč?", is yes.** The action of the operators $\tilde{a}^a$ on the basic functions $\psi_{\alpha\beta}(\{\theta^a\})$ transforms the index α, keeping the index β fixed. The action of the operators $\tilde{\tilde{a}}^a$ on the basic functions $\psi_{\alpha\beta}(\{\theta^a\})$ transforms the index β, keeping fixed the index α. In both cases the basic functions transform as spinors and accordingly fulfill the Dirac equation.
The Lorentz transformations are in the Mankoč approach determined either only with $\tilde{a}^a$ ($\tilde{S}^{ab} = \frac{i}{4}[\tilde{a}^a, \tilde{a}^b]$) or only with $\tilde{\tilde{a}}^a$ ($\tilde{\tilde{S}}^{ab} = \frac{i}{4}[\tilde{\tilde{a}}^a, \tilde{\tilde{a}}^b]$). In the Dirac-Kähler case the Lorentz transformations transform either the index α or the index β. If one shifts what is meant by a Lorentz transformation, then of course it is not so surprising, if it turns out that there can appear particles/states with an a priori unexpected spin.
Not only we have establish the connection between these two approaches, **we have also shown that the Dirac matrices as appear in the Dirac-Kähler approach have a Grassmann odd character. To make them having Grassmann even character the transformation of the index α should be accompanied by the simultaneous transformation of the index $\beta = 0$.** We have also learned that the Dirac-Kähler approach to spinors have for odd dimensional spaces mixed Grassmann character. Accordingly it is the appropriate superposition of the Dirac-Kähler basis which can be used to describe spinors.
Since in both cases, that is in the Mankoč approach and the Dirac-Kähler approach, the dimension of the space is 2^d, it **means that in the four dimensional space-time there are four times four spinors, which may be responsible for four families of quarks and leptons.** This four flavour prediction follows **both** in Norma's and in other uses of the Dirac-Kähler - like approach. Both suffer from the empirical evidence for only three generations.

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Comments on the Hierarchy Problem

Berthold

The standard model treated with a momentum space cut-off has quadratic and logarithmic divergencies. In perturbation theory fine-tuned large subtractions depending precisely on the standard model parameters have to be performed. This is quite unnatural and known as the hierarchy problem. How the scale of weak interaction and, in particular, how scalar masses are protected, is not understood. Quadratic divergencies make severe problems in higher-order calculations. They can formally be avoided by using dimensional regularization. They do not occur in supersymmetric models where the supersymmetric partners of each particle provide for a cancellation of the quadratic divergence. In spite of this elegant solution, crude attempts have been made and are still made to obtain a full or near cancellation of divergencies by special choices of the standard model couplings which could make the introduction of super partners unnecessary [1],[2].

The divergencies can best be studied by looking at the logarithm of the partition function, the "free energy" $W(J,j)$ [3]. Here J denotes a parameter in the Higgs Lagrangian of dimension m^2 which multiplies the square of the Higgs field. j describes a field coupled linearly to the neutral Higgs component. The vacuum expectation value of the unrenormalized neutral Higgs field is obtained from

$$< \Phi_0 > \sim \frac{\partial W(J,j)}{\partial j}\Big|_{J=J_0, j=0} \quad . \tag{6.1}$$

We denoted the unrenormalized mass term in the action by J_0. The vacuum expectation value of the square of the unrenormalized Higgs field can be found from

$$< \Phi^\dagger \Phi > \sim \frac{\partial W(J,j)}{\partial J}\Big|_{J=J_0, j=0} \quad . \tag{6.2}$$

The calculation of $W(J,j)$ to one-loop order can be done by the saddle-point method. Taking a universal cut-off Λ for all propagators, the quadratic divergence arising from (6.1) is proportional to

$$S_{\Lambda^2} = (3m_H^2 + 3m_Z^2 + 6m_W^2 - 12\sum_f m_f^2)/ < \Phi_0 >^2 \tag{6.3}$$

For simplicity the couplings contained in (6.3) are given in terms of the masses. The sum in (6.3) extends over all fermions which obtain their masses through their couplings to the Higgs field. The same expression for S_{Λ^2} is obtained from (6.2) after subtracting the free field divergency as discussed in reference [3].

The vanishing or smallness ($\sim 1/\Lambda^2$) of S_{Λ^2} has been proposed to diminish the hierarchy problem. The corresponding mass relation is usually referred to as the Veltman condition. It gave a lower limit on the top quark mass before the top was discovered. One of the problems with the Veltman condition is the question of the scale at which $S_{\Lambda^2} \simeq 0$ should hold. The vacuum expectation value of the unrenormalized Higgs field is scale-independent, but higher-order calculations and the knowledge of the scale dependence of Λ are needed (Λ may be related to high mass states) to take advantage of this fact. So far it remains open which scale μ should be taken such that higher order terms are reasonably small and the effect of the cut-off is suppressed. Because of the strong scale dependence of the top quark Yukawa coupling according to the renormalisation group equations the predictions for the Higgs mass from the requirement $S_{\Lambda^2} = 0$ ranges from $m_H(m_Z) \approx 320$ GeV for $\mu \approx m_Z$ down to $m_H(m_Z) \approx 140$ GeV for $\mu \approx m_{Planck}$.

Let us now have a look at the logarithmic dependence on the cut-off Λ. There is a difference between the results obtained from the vacuum expectation value of the linear Higgs field (6.1) and the one from the square of the Higgs field (6.2). In the first case the $\log \Lambda$ term is governed by the combination [2]

$$S^L_{Log\Lambda} = (\frac{3}{2}m_H^4 + 3m_Z^4 + 6m_W^4 - 12\sum_f m_f^4)/ < \Phi_0 >^4 \quad . \tag{6.4}$$

We note that $W(J_0,j)$ for $j \neq 0$ is gauge-dependent, and so is (6.4) as well as the Higgs potential $V(J, < \phi_0 >)$ obtained by the Legendre transformation of $W(J,j)$ with respect to the variable j. Here, the Landau gauge seems to be the most appropriate one [2]. For the square of the Higgs field, on the other hand, the corresponding expression takes the same form as in (6.4) but with a different coefficient multiplying m_H^4 [3]

$$S^{SQ}_{Log\ \Lambda} = (\zeta m_H^4 + 3m_Z^4 + 6m_W^4 - 12\sum_f m_f^4)/ < \Phi_0 >^4 \quad . \tag{6.5}$$

The parameter ζ distinguishes two cases due to a subtraction term $\sim J^2$ in $W(J,j)$ which contributes here but does not contribute to $< \Phi_0 >$ obtaineable from (6.1). $\zeta = 1$ follows if no subtraction is performed. Presumably, however, one should take $\zeta = 0$ which is obtained by fixing the subtraction term such that the logarithmic divergence occuring in (6.2) vanishes in the unbroken phase.

It is not obvious whether (6.1) or (6.2) is the better choice for discussing logarithmic divergencies. The vacuum expectation value of the square of the Higgs field is gauge-invariant and may be prefered.

To require $S_{Log\ \Lambda} \simeq 0$ is of course again a very speculative assumption. The one-loop results (6.4) and (6.5) are strongly scale-dependent. We set $m_t(m_Z) = 173\ GeV$ and use the two loop renormalisation group equations for the calculation of coupling constants at other scales. By taking the scale μ from $\mu \approx m_Z$ up to $\mu \approx m_{Planck}$ one obtains $m_H(m_Z)$ values ranging from ≈ 290 GeV down to ≈ 150 GeV if using $S^L_{Log\ \Lambda} = 0$ (eq. (6.4)), and ≈ 320 GeV down to ≈ 120 GeV by using $S^{SQ}_{Log\ \Lambda} = 0$ (eq. (6.5)) with $\zeta = 1$. $\zeta = 0$ requires a very large scale of the order of the Planck scale (together with a top quark mass $m_t(m_Z) \approx 170\ GeV$) (see below).

We will now consider the simultaneous suppression of quadratic and logarithmic divergencies. We then obtain – at a given scale – two equations containing the Higgs and top coupling constants. There are two solutions, one with large values for the Higgs and top masses and one with much smaller values. We first consider the one with the larger values and choose the scale μ such that $m_t(m_Z) = 173\ GeV$. The result is now different whether $S^L_{Log\ \Lambda}$ or $S^{SQ}_{Log\ \Lambda}$ is used. In the case of $S^L_{Log\ \Lambda} = 0$ suggested in ref. [2] one gets $m_H(m_Z) \approx$ 178 GeV. The corresponding scale μ, which can be interpreted as the cut-off scale, is found to be $\approx 10^4\ TeV$.

If we require the vanishing of $S^{SQ}_{Log\ \Lambda}$ together with S_{Λ^2} and take $\zeta = 1$ one obtains $m_H(m_Z) \approx 330\ GeV$ and $m_t(m_Z) \approx 200\ GeV$ when choosing for the scale the very low value $\mu \approx 250\ GeV$. This case is interesting since $m_H \approx 2m_t$ suggests a bound state picture for the Higgs meson. The low scale could mean that the influence of higher states on the standard model would be noticeable already in the low TeV region. Above this scale the renormalization-group equations of the standard model would no more be valid. Of course, it could be entirely fortuitous that $m_H \approx 2m_t$ taken at a low scale leads to small values for S_{Λ^2} and $S^{SQ}_{Log\ \Lambda}$, simultaneously. We also note that $\zeta = 0$ does not lead to admissable solution at this low scale.

For the second type of solutions with the smaller values for the Higgs and top couplings the relevant scale μ must be very high, as high as the Planck mass, in order to obtain a value of $m_t(m_Z)$ near 170 GeV. Let us then take the scale to be equal to the Planck scale $\mu = 10^{19}\ GeV$. Then we can "predict" both, the top and the Higgs mass. Because of the smallness of the Higgs coupling it turns out, that the result does not much differ whether we use (6.3) together with (6.4) or (6.3) with (6.5) and $\zeta = 1$ or with (6.5) and $\zeta = 0$. One obtains

$$m_t(m_Z) \approx 170\ GeV\ ,\ m_H(m_Z) \approx 140\ GeV\ . \tag{6.6}$$

This result is very close to the one obtained by Bennett, Nielsen and Froggatt [4] using a different reasoning. It is close since these authors are also led to $S^{SQ}_{Log\ \Lambda} = 0$ at the Planck scale. There is no precise agreement since the zero value of the Higgs coupling at the Planck mass they argue for does not occur in the approach presented here.

The solution obtained by combining $S_{\Lambda^2} = 0$ with $S_{Log\ \Lambda} = 0$ and taking $\zeta = 0$ is of particular interest. First, it requires a very high scale which can be identified with the cut-off Λ. This scale is large enough that even the logarithm of it can provide for an order of magnitude suppression of new physics. Secondly, the divergencies are eliminated in the unbroken phase as well. Thirdly, (6.5) can be written in terms of β - functions for the renormalized Higgs coupling $\lambda(\mu)$ and the renormalized mass parameter in the Higgs Lagrangian $J_0(\mu)$

$$4\pi^2\ S^{SQ}_{Log\ \Lambda} = -\frac{\lambda^2(\mu)}{J_0^2(\mu)}\beta(\frac{J_0^2(\mu)}{\lambda(\mu)})\ \ . \tag{6.7}$$

Thus, the requirement that (6.7) vanishes can formally be viewed as a condition for a "fix point" at $\mu \approx m_{Planck}$ even though the full vacuum expectation value of the square of the unrenormalized Higgs field is scale independent. Taking $\mu = 10^{19}\ GeV$ and $\alpha_s = 0.12$ one gets

$$m_t(m_Z) = 168\ GeV\ ,\ m_H(m_Z) = 137\ GeV\ . \tag{6.8}$$

As a last point I like to comment on the question of the possible participation of a 4th generation. If the particles of this generation obtain their masses through the coupling to the standard model Higgs particle, one cannot have S_{Λ^2} and $S_{Log\ \Lambda}$ simultaneously sufficiently small or zero. The reason is that fermion masses for the b' and t' would enter with masses which are larger or roughly equal to the mass of the top. One obtains imaginary solutions of the equations or – for $\zeta = 0$ – too small values for the fermion masses.

We have seen that within the standard model the possibility exists that the couplings are arranged such that the influence from physics beyond the standard model is suppressed. This is not a trivial statement.

Furthermore, it can only occur for the known three generations[5]. When we minimize simultaneously the quadratic and the logarithmic dependence of the vacuum expectation value of the Higgs field on the cut-off we find interesting relations between the Higgs and the top mass. For a very low value of the cut-off scale we found $m_H \approx 2m_t$. On the other hand, when using a properly subtracted form of $W(J, j)$ such that there is no divergence in the parameter region of no symmetry breaking, the required absence of quadratic and logarithmic divergencies in the physical region leads to $m_H \approx 140\ GeV$.

I like to thank Norma Mankoc Borstnic and Holger Bech Nielsen for inviting me to this workshop. I profited from the many discussions, especially also with Colin Froggatt.

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Parity conservation in broken $SO(10)$

Hanns

Nowadays, it is obvious that the suitable framework of discussing particle physics is given by spontaneously broken gauge theories. Therefore, also discrete symmetries like P, CP and T have to be implemented into this framework [1]. Generally, parity may be broken if left- and righthanded fermions belong to different representations of the gauge Group G. In vector like gauge theories, like QCD and QED, parity is trivially conserved since in both cases the left- and righthanded fermions belong to the same fundamental representation of $SU(3)_C$, or $U(1)_{EM}$, respectively. Some gauge theories, like $SO(10)$, however allow a more subtle definition of parity (called internal parity [1], or D-parity [2,3]) as a special automorphism of the Lie algebra even if the representation of left chiral fermions is different from the right chiral one. In such a theory, parity violation is linked to the spontaneous breakdown of the corresponding Grand Unified Theory.

For simplicity, we write all fermions of one family as lefthanded fields

$$f_L = (\psi_{1L}, (\psi_2^c)_L = C\bar{\psi}_{2R}^T).$$

In $SO(10)$, all 16 quarks and leptons (including a righthanded neutrino ν_R) of one family, written as lefthanded fields, belong to one irred. 16 representation:

$$f_L \sim 16, \qquad f_R \sim \overline{16}.$$

For convenience, we use the following subgroup of $SO(10)$ for the spectrum of states:

$$SO(10) \supset SO(4) \otimes SO(6) \simeq SU(2)_L \otimes SU(2)_R \otimes SU(4)_C.$$

Here, $SU(4)_C$ is the Pati–Salam color group [4] with lepton number as fourth color, i.e. $SU(4)_C \supset SU(3)_C \otimes U(1)_{B-L}$. With this, we find

$$16 = (2,1,4) + (1,2,\bar{4}).$$

One can define a parity transformation via

$$P : (f_i)_L \to U_{ij}(P)(f_j^c)_L.$$

Since f_{iL} and f_{iL}^c are both members of the 16-representation, the matrix $U(P)$ can be expressed in terms of the $SO(10)$-generators M_{ij} as follows [1]:

$$U(P) = \exp(i\pi M_{46} \cdot M_{810}).$$

In most models of $SO(10)$ [5,6], the breaking pattern to the standard model follows the chain

$$\begin{aligned} SO(10) &\to SU(2)_L \otimes SU(2)_R \otimes SU(4)_C \otimes P \\ &\to SU(2)_L \otimes SU(2)_R \otimes SU(3)_C \otimes U(1)_{B-L} \otimes P \\ &\to SU(2)_L \otimes U(1)_Y \otimes SU(3)_C \end{aligned}$$

or alternatively

$$\begin{aligned} SO(10) &\to SU(2)_L \otimes SU(2)_R \otimes SU(4)_C \otimes P \\ &\to SU(2)_L \otimes U(1)_R \otimes SU(4)_C \\ &\to SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L} \otimes SU(3)_C \\ &\to SU(2)_L \otimes U(1)_Y \otimes SU(3)_C. \end{aligned}$$

In both cases, the breaking of parity (P) is associated with the breaking of $SU(2)_R$ and is therefore characterized by the same scale Λ_R, $\Lambda_R > 350$ TeV. There exists, however, an alternative scenario, given by Chang et al. [3] where parity breaks separately from $SU(2)_R$ at a much higher scale:

$$\begin{aligned} SO(10) &\to SU(2)_L \otimes SU(2)_R \otimes SU(4)_C \otimes P \\ &\to SU(2)_L \otimes SU(2)_R \otimes SU(4)_C \\ &\to SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L} \otimes SU(3)_C \\ &\to SU(2)_L \otimes U(1)_Y \otimes SU(3)_C. \end{aligned}$$

The reason for the last breaking chain is the assumption of the existence of a real P-odd $SU(2)_R$ singlet Higgs field which acquires a vacuum expectation value at a high scale, independent of Λ_R. It can be incorporated in a 210-dim. Higgs representation, whereas the usual breaking pattern proceeds via 120- and 126-dimensional representations only.

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1 Discussion on Majorana particles

What is the Majorana propagator? A Majorana spinor field $\psi_M(x)$ is a four component spinor satisfying the additional constraint

$$\psi_M = \psi_M^c \equiv C\bar{\psi}_M^T,$$

and

$$\bar{\psi}_M = \overline{\psi_M^c} \equiv -\psi_M^T C^{-1}$$

(for anticommuting fields). Written in terms of two-component chiral Weyl spinors [1], a Majorana spinor is given by

$$\psi_M = \left(\begin{array}{c} \chi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{array} \right)$$

whereas a Dirac spinor is written as

$$\psi_D = \left(\begin{array}{c} \chi_\alpha \\ \bar{\lambda}^{\dot{\alpha}} \end{array} \right).$$

Correspondingly, the usual Dirac invariant and Majorana invariant are identical:

$$\bar{\psi}\psi = \bar{\psi}^c\psi = -\psi^T C^{-1}\psi.$$

Furthermore, the Dirac equation for a Majorana particle reads, in the Weyl representation where γ_5 is diagonal,

$$\left(\begin{array}{cc} 0 & i\sigma^\mu\partial_\mu \\ i\bar{\sigma}_\mu\partial^\mu & 0 \end{array} \right) \left(\begin{array}{c} \chi \\ \bar{\chi} \end{array} \right) = m \left(\begin{array}{c} \chi \\ \bar{\chi} \end{array} \right)$$

and the Lagrangian is given by

$$\mathcal{L} = \frac{i}{4}\bar{\psi} \overleftrightarrow{\not\partial} \psi - \frac{m}{2}\bar{\psi}\psi.$$

Note the additional factor 1/2, which is due to the fact that $\bar{\psi}$ is proportional to ψ. Decomposing the field operator into creation and annihilation operators a_s, $a_s^\dagger$ and b_s, $b_s^\dagger$ and using $\psi^c = C\bar{\psi}^T$, one finds that $a = b$ and therefore

$$\psi_M(x) = \int \frac{d^3k}{(2\pi)^3 2k_0} \left\{ \sum_s a_s(k)u_s(k)e^{-ikx} + \sum_s a_s^\dagger(k)v_s(k)e^{ikx} \right\}$$

where

$$\{a_s(k), a_{s'}^\dagger(k')\}_+ = (2\pi)^3 2k_0 \delta^3(\vec{k} - \vec{k}').$$

¿From this, it is straightforward to define the propagator functions for Majorana particles [2]

$$\langle 0|T(\psi_M(x), \bar{\psi}_M(x')|0\rangle = iS_F(x - x')$$

which is identical to

$$\langle 0|T(\psi_M(x), \psi_M(x')|0\rangle = iS_F(x - x')C^T$$

$$\langle 0|T(\bar{\psi}_M(x), \bar{\psi}_M(x')|0\rangle = iC^{-1}S_F(x - x').$$

With this, it is straightforward to derive the Feynman rules. The only difference to Dirac particles one should be aware of is that internal Majorana particles have no arrow which indicates the flow of fermion number.

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2 Representations of $SO(1,13)$

Here, I will give a short review of the fundamental spinor representations of $SO(1,13)$ in terms of the Lorentz group $\otimes SO(10)$, as grand unification group. Details can be found in Ref. [1].

The two fundamental representations of $SO(1,13)$ are the spinor representations $S^+ = 64$ and $S^- = 64'$. According to the subgroup $SO(1,3)\otimes SO(10)$, and denoting the representations of the Lorentz group by $\mathcal{D}(j,j')$, we find

$$64 = \mathcal{D}(1/2,0)\otimes 16 + \mathcal{D}(0,1/2)\otimes \overline{16} = (16)_L + (\overline{16})_R,$$

$$64' = \mathcal{D}(1/2,0)\otimes \overline{16} + \mathcal{D}(0,1/2)\otimes 16 = (\overline{16})_L + (16)_R.$$

This means, the 64-representation contains exactly the desired states of one family of fermions, a 16-representation of left chiral fermions and a $\overline{16}$ of their right chiral antiparticles.

The $64'$, however, has the particle content exactly reversed – $\overline{16}$ for the left chiral fields, 16 for their antiparticles. Such a family would consist exclusively of mirror-particles, i.e. left chiral quarks which are singlets and right chiral quarks which are doublets under $SU(2)_L$. It should be noted, due to the signature of $SO(1,13)$, both reps are real and inequivalent, i.e.

$$64 = \overline{64}, \qquad 64' = \overline{64}' \neq 64.$$

If we are looking for gauge bosons V^μ of $SO(1,13)$, they should be contained in the (adjoint) 91-representation:

$$91 = \mathcal{D}(1,0+0,1)\otimes 1 + \mathcal{D}(1/2,1/2)\otimes 10 + \mathcal{D}(0,0)\otimes 45.$$

The first term denotes the usual affine connections ${\Gamma^\mu}_{\nu\lambda}$, the gauge bosons of the Lorentz group. The last term contains the familiar 45 gauge bosons ${V^\mu}_{ij}$ of $SO(10)$, whereas the second term mixes Lorentz- and internal degrees of freedom. Its coupling to the fermions in 64 looks like the coupling of a spin-2 tensor meson in the 10-dimensional representation of $SO(10)$.

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